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# Effect analysis of the general complex reciprocal gyro-bianisotropic metamaterial medium on the input impedance of a printed dipole antenna

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 Method of moment

**Abstract** The aim of this paper is to present an analytical study for the investigation of the reciprocal metamaterial gyro-bianisotropy effects on the input impedance of a dipole antenna. This study is based on the numerical resolution, using the spectral method of moments (SMoM), of the integral equation developed through the mathematical derivation of the appropriate spectral Green's functions of the studied dipole configuration. The original obtained results are discussed and showed good agreements with the isotropic studied cases available in published literature. It is found that the imaginary valued magnetoelectric elements (chiral medium) exhibit an inverse effect on the resonant length compared to purely real elements (Tellegen medium), while negative

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elements decrease the input impedance. Combination of the two cases has led to more general complex media with further inspiring features.

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## 1. Introduction

Recently, as material sciences have significantly advanced, the artificial media, such as chiral and general bianisotropic materials, have gained increased interest from researchers and industrials in the field of microwaves and optics for their unusual and exciting properties [1–5].

Several research works have dealt with the study of antenna structures based on chiral materials using different techniques. In [1,2], a microstrip antenna and a multi-element strip antenna array implanted on a chiral metamaterial substrate are investigated, respectively. Both studies are based on the derivation of the Cauchy-singularity integral equation and the antenna-length-dependent input impedance is presented. In [2], it is shown that chiral metamaterial substrates present potential solutions for mutual coupling reduction in antenna arrays, where a system of singular integral equations based on Bessel functions and Chebyshev polynomials is derived. In [1], the effect of chiral substrates on a two-element antenna array input impedance is presented, where a decrease in the quality factor and a shortening effect are noticed. In [4], a metamaterial microstrip antenna MIMO system is presented. It is stated that such substrates reduce the mutual coupling between the emitters and the use of chiral substrates with fractal strips improves their characteristics. In [1–11], the resonant frequency, bandwidth, input impedance or mutual coupling of dipole antennas printed on anisotropic and bi-isotropic chiral materials have been investigated. Bianisotropic boundaries and their scattering features have been considered for study since the late 1980 s including that by M. M. Idemen [12]. C. L. Holloway and E. F. Kuester's works in the early 2000 s [13–15] have revived interest in this field. However, only in the past few years the true potential of bianisotropic metasurfaces has been revealed by researchers [16].

In this paper, we present an analytical study of a dipole antenna printed on a substrate with a reciprocal gyro-bianisotropy, where the magnetoelectric elements of the constitutive parameters are complex valued. Particularly, the effect of the complex-valued elements on the input impedance, resonant length is investigated. The study is based on a spectral theoretical formulation and a numerical solution technique using the SMoM [3,5–11].

The originality of this work is the choice of purely real magnetoelectric parameters (Tellegen case), a case that has never been applied in antenna design [11].

It is worth noting that the reciprocal gyro-bianisotropic metamaterial substrate opens up a wide range of applications. Research in this area provides further opportunities in application areas. The additional degrees of freedom offered by bianisotropic materials promise conformal microwave and optical systems that can be appropriately integrated into a variety of electronic structures. These include ultra-thin and flat panel antennas with arbitrary aperture distributions [17–20], compact mode converters [21], transitions and couplers, conformal

cloaking membranes [22,23], as well as ultra-thin cameras, detectors, and high-resolution 3D holographic displays [16]. Alternatively, nonreciprocal bianisotropic metasurfaces can be used to design, for instance, various types of isolators with simultaneous control of amplitude and polarization of the propagating waves [24].

The complex form of the gyrotropic parameters allowed us to control and adjust the input impedance peaks without altering the resonant frequency compared to the isotropic case. Introduction of chirality in the substrate opens a wide range of applications, and definitely its major effect is the aptitude for structure miniaturization.

## 2. Analytical formulation

The corresponding constitutive relations of a general complex bianisotropic medium are expressed, in their general form [25,26]:

$$\vec{D} = [\epsilon] \vec{E} + \sqrt{\epsilon_0 \mu_0} [\eta] \vec{H} \quad (1a)$$

$$\vec{B} = [\mu] \vec{H} + \sqrt{\epsilon_0 \mu_0} [\xi] \vec{E} \quad (1b)$$

where the permittivity  $[\epsilon]$ , the permeability  $[\mu]$  and the magnetoelectric parameters  $[\xi]$  and  $[\eta]$  are  $3 \times 3$  tensors.

Chiral materials can be realized using a bi-isotropic medium with randomly distributed right- and left-handed helices (spirals) or bianisotropic based on orderly distributed right- and left-handed helices (cf. Fig. 1 [4]). Several configurations of such structures were presented in [4], in particular, the structures with vertically or horizontally oriented helices (cf. Fig. 2a and b). In [25], another type of a bianisotropic metamaterial is designed using Split Ring Resonators (SRRs), schematically shown by its unit cell (cf. Fig. 1d). The split ring acts as a LC circuit, where the loop and gap are equivalent to an inductor and a capacitor, respectively. The bianisotropic property can be described by the fields and current distribution for a z-polarized incident wave, as reported in the literature [25,26].

In the present work, we consider for study a general gyro-bianisotropic medium characterized by  $3 \times 3$  permittivity, permeability and magnetoelectric tensors. Assuming that the cou-

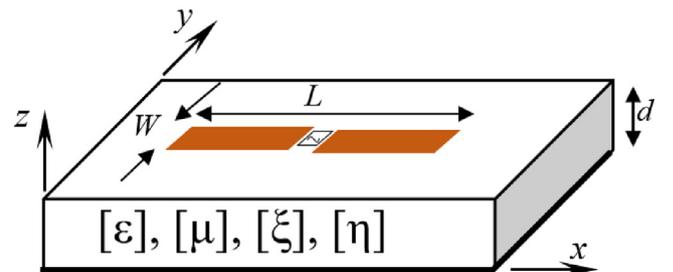
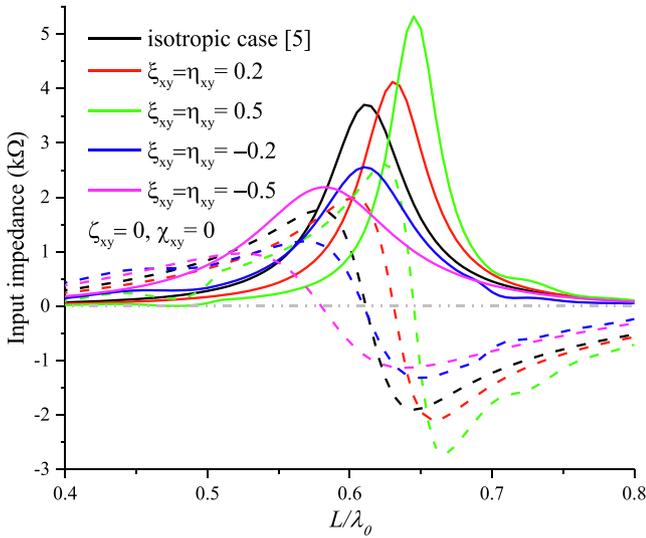


Fig. 1 Geometries of Printed dipole.



**Fig. 2** Effects of positive and negative reciprocal chirality on the input impedance of the printed dipole.

pling between the magnetic and electric fields only exists in the x-y plane. These tensors take the following forms:

$$[\epsilon] = \epsilon_0 \begin{bmatrix} \epsilon_t & 0 & 0 \\ 0 & \epsilon_t & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \quad (2a)$$

$$[\mu] = \mu_0 \begin{bmatrix} \mu_t & 0 & 0 \\ 0 & \mu_t & 0 \\ 0 & 0 & \mu_z \end{bmatrix} \quad (2b)$$

$$[\xi] = \begin{bmatrix} 0 & (\chi_{xy} + j\xi_{xy}) & 0 \\ -(\chi_{xy} + j\xi_{xy}) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (2c)$$

$$[\eta] = \begin{bmatrix} 0 & (\varsigma_{xy} + j\eta_{xy}) & 0 \\ -(\varsigma_{xy} + j\eta_{xy}) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (2d)$$

Generally, the magnetoelectric coupling effects can be classified as reciprocal or non-reciprocal. In reciprocal media, the permittivity and permeability dyadics are symmetric, and the two magnetoelectric dyadic coefficients are related as  $[\eta] = -[\xi]^T$ , only one magnetoelectric dyadic is sufficient to describe the nonreciprocal coupling [27].

The non-reciprocity parameters  $\chi_{xy}$  and  $\varsigma_{xy}$  are needed to model natural magnetoelectric effect which occurs, for example, in some ferromagnetic and anti-ferromagnetic crystals. Recently, it is suggested how such media can be realized as artificial composites for microwave applications [28]. Because these tensors are real valued, the Tellegen response breaks time reversal symmetry, distinguishing it from other bianisotropic electromagnetic responses (such as SRRs and chiral metamolecules), which do not [29]. As is commonly accepted, we use the name chirality parameter for the coupling  $j\xi_{xy}$  and  $j\eta_{xy}$  in bianisotropic reciprocal media [28]. In what follows,

we investigate the following three cases according to the conditions of consideration given below [27,28].

- $[\xi] = -[\eta]^T = [\eta] = \begin{bmatrix} 0 & j\xi_{xy} & 0 \\ -j\xi_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$  (reciprocal chiral)
- $[\xi] = -[\eta]^T = [\eta] = \begin{bmatrix} 0 & \chi_{xy} & 0 \\ -\chi_{xy} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$  (reciprocal Tellegen)
- $[\xi] = -[\eta]^T = [\eta] = \begin{bmatrix} 0 & (\chi_{xy} + j\xi_{xy}) & 0 \\ -(\chi_{xy} + j\xi_{xy}) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$  (reciprocal complex bianisotropic)

The general planar dipole antenna geometry and the associated coordinate system, with the optical axis  $oz$  as direction of propagation, are illustrated in Fig. 1. The presented configuration is used to investigate the effect of reciprocal gyro-bianisotropy on the input impedance of the printed dipole (Fig. 1.a) and to evaluate the mutual coupling between two-dipole antenna array (Fig. 1.b).

The considered material medium can be realized with complex imaginary-valued elements of the constitutive parameters. This can be done by inserting a periodic set of inclusions in a dielectric substrate. Realistic experimental models of this bianisotropic metamaterial is presented in [25]. The simplest cases of this medium have been used as a substrate of a dipole antenna by Sayad *et al.* [10], where the magnetoelectric elements are equal and purely imaginary. The case of non-equal magnetoelectric elements was studied by Zebiri *et al.* in [30]. The results showed that the contribution of the elements  $\xi$  and  $\eta$  in the calculation of the input impedance is  $\frac{1}{2}(\xi + \eta)$  in the dyadic Green's function described by Equ. 20 in [30]. On the other hand, its contribution in the electromagnetic field components is denoted by the factor  $e^{-\kappa_0 \frac{1}{2}(\xi - \eta)z}$ , in Eqs. (9) and (10) [30], that expresses gain or loss, depending on the choice of  $\xi$  and  $\eta$  values.

In [10,31], only cases of media with imaginary valued elements have been investigated, *i.e.*, the case of reciprocal chiral media ( $[\xi] = -[\eta]^T$ ). In this work, we examine a more complex issue which consider this novelty of cases that have never been investigated.

The expected waves propagating in a grounded dielectric slab are surface wave modes which are either TE or TM with respect to the interface normal. The longitudinal components  $E_z$  and  $H_z$  are found to satisfy two decoupled homogeneous second-degree differential wave equations:

$$\frac{\partial^2 \tilde{E}_z}{\partial z^2} - \Gamma_1 \frac{\partial \tilde{E}_z}{\partial z} + \Gamma_2 \tilde{E}_z = 0 \quad (3a)$$

$$\frac{\partial^2 \tilde{H}_z}{\partial z^2} - \Gamma_1 \frac{\partial \tilde{H}_z}{\partial z} + \Gamma_3 \tilde{H}_z = 0 \quad (3b)$$

where

$$\Gamma_1 = j\kappa_0(\varsigma_{xy} - \chi_{xy} + j(\eta_{xy} - \xi_{xy})) \quad (3c)$$

$$\Gamma_2 = \left( \left( \kappa_0^2 \epsilon_t \mu_t - \frac{\epsilon_t}{\epsilon_z} (\alpha^2 + \beta^2) \right) + \kappa_0^2 (\chi_{xy} + j\xi_{xy})(\varsigma_{xy} + j\eta_{xy}) \right) \quad (3d)$$

$$\Gamma_3 = \left( \left( \kappa_0^2 \epsilon_t \mu_t - \frac{\mu_t}{\mu_z} (\alpha^2 + \beta^2) \right) + \kappa_0^2 (\chi_{xy} + j\zeta_{xy}) (\varsigma_{xy} + j\eta_{xy}) \right) \quad (3e)$$

The particularity of the general medium is pronounced by the extra term  $\Gamma_1 \frac{\partial \tilde{E}_z}{\partial z}$  in the second-order differential wave Eqs. (3a) and (3b). This additional term can be interpreted by a loss or a gain in the amplitude of the electromagnetic fields [30], which is reminiscent of the Schrodinger equation for an electron in presence of a magnetic potential [32].

### 3. METHOD OF SOLUTION

Solving the two differential Eqs. (3a) and (3b), for  $\tilde{E}_z$  and  $\tilde{H}_z$  in the dielectric region, gives:

$$\tilde{E}_z(\gamma_e, z) = e^{\kappa_0 \frac{(\xi_{xy} - \eta_{xy})}{2} z} e^{j\kappa_0 \frac{(\varsigma_{xy} - \chi_{xy})}{2} z} (A_e \cosh(\gamma_e z) + B_e \sinh(\gamma_e z)) \quad (4a)$$

$$\tilde{H}_z(\gamma_h, z) = e^{\kappa_0 \frac{(\xi_{xy} - \eta_{xy})}{2} z} e^{j\kappa_0 \frac{(\varsigma_{xy} - \chi_{xy})}{2} z} (A_h \sinh(\gamma_h z) + B_h \cosh(\gamma_h z)) \quad (4b)$$

where  $A_e$ ,  $B_e$ ,  $A_h$  and  $B_h$  are complex constants and

$$\gamma_e^2 = \left( \frac{\epsilon_t}{\epsilon_z} (\alpha^2 + \beta^2) - \kappa_0^2 \epsilon_t \mu_t \right) - \gamma_c^2 \quad (4c)$$

$$\gamma_h^2 = \left( \frac{\mu_t}{\mu_z} (\alpha^2 + \beta^2) - \kappa_0^2 \epsilon_t \mu_t \right) - \gamma_c^2 \quad (4d)$$

$$\gamma_c^2 = \left( \frac{\kappa_0}{2} ((\varsigma_{xy} + \chi_{xy}) + j(\eta_{xy} + \zeta_{xy})) \right)^2 \quad (4e)$$

For  $\varsigma_{xy} = \chi_{xy} = 0$ , the expressions (4a)–(4e) are the same found in [30]. Thus, we find that the solution for a plane EM wave in a complex gyro bianisotropic medium consists of a nonreciprocal  $z^+$ -directed propagating wave and a  $z^-$ -directed one with  $e^{\kappa_0 \frac{(\xi_{xy} - \eta_{xy})}{2} z} e^{j\kappa_0 \frac{(\varsigma_{xy} - \chi_{xy})}{2} z}$ . For the nonreciprocal achiral this term will be  $e^{\kappa_0 \frac{(\xi_{xy} - \eta_{xy})}{2} z} e^{j\kappa_0 \frac{(\varsigma_{xy} - \chi_{xy})}{2} z}$ . There may be a gain in one direction and a loss in the other. The solution resembles that of a lossy dielectric medium only in one direction and just for one medium ( $\eta_{xy} \neq \zeta_{xy}$ ,  $\varsigma_{xy} \neq -\chi_{xy}$ ). This is a very interesting feature that has to be well considered. Because in these conditions the medium behaves like an isotropic dielectric with the presence of the term  $e^{\kappa_0 \frac{(\xi_{xy} - \eta_{xy})}{2} z} e^{j\kappa_0 \frac{(\varsigma_{xy} - \chi_{xy})}{2} z}$ . Therefore, The non-reciprocal medium (Tellegen or achiral) contributes by a  $e^{j\kappa_0 \frac{(\xi_{xy} - \eta_{xy})}{2} z}$  or  $e^{\kappa_0 \frac{(\xi_{xy} - \eta_{xy})}{2} z}$ , respectively, in the solution which must be deeply examined in future works.

By applying the boundary conditions, the spectral expressions of the electric and magnetic tangential components are evaluated at the interface air-dielectric in terms of the tangential current densities on the strips  $\tilde{J}_x$  and  $\tilde{J}_y$  [3,10,11]. A matrix of the Green's tensor elements  $G_{ij}$  for the dipole structure is formulated in [10,30].

In the analysis of narrow dipole configurations, the cross-current density in the  $y$ -direction is commonly ignored, as it is assumed that the width of the dipole is negligible [10]. Con-

sequently,  $\tilde{G}_{xx}$  is the only presented Green's function, since the others are not involved in the calculations. The function  $\tilde{G}_{xx}$  for this medium is derived and is given by:

$$G_{xx} = \frac{-j}{\omega \epsilon_0 (\alpha^2 + \beta^2)} \left[ \frac{z^2 \gamma_0 (\gamma_e^2 + \gamma_h^2)}{\gamma_0 \epsilon_t \gamma_e \coth(\gamma_e d) + ((\gamma_e^2 + \gamma_h^2) - j\gamma_0 \epsilon_t \gamma_c)} - \frac{\beta^2 \kappa_0^2 \mu_t}{(\gamma_h \coth(\gamma_h d) + \mu_t \gamma_0 - j\gamma_c)} \right] \quad (6a)$$

Magnetolectric-depending sub-cases of this general bianisotropic medium can be verified. For  $\chi_{xy} = \varsigma_{xy} = 0$ , the Green's tensor expression is the same as that found in [30], for  $[\zeta] = [\eta] \neq 0$ , we find the expression derived in [10] and for a dielectric with a uniaxial anisotropy  $[\zeta] = [\eta] = 0$ , we obtain the same medium and expressions as that treated in [3,5].

### 4. Numerical results

In this work, we are interested in the investigation of the effect of the reciprocal gyro-bianisotropic metamaterial substrate on the input impedance of the printed dipole antenna. Before discussing the results of this study, we point out that a validation of the method and the solution technique was carried out in [3,5] by comparing with literature.

#### A. Effect of the reciprocal chiral ( $\xi_{xy} = \eta_{xy}$ ) on the input impedance and the resonance frequency

Fig. 2 illustrates the variation of the input impedance of a reciprocal chiral dipole antenna for different positive and negative values of  $\xi_{xy}$  and  $\eta_{xy}$  with a permittivity of  $\epsilon_r = 3.25$  and a permeability of  $\mu_r = 1$ , compared to the isotropic case medium. In this case, the effect of the parameters  $\xi_{xy}$  and  $\eta_{xy}$  is reciprocal, neither on the shape of the input impedance.

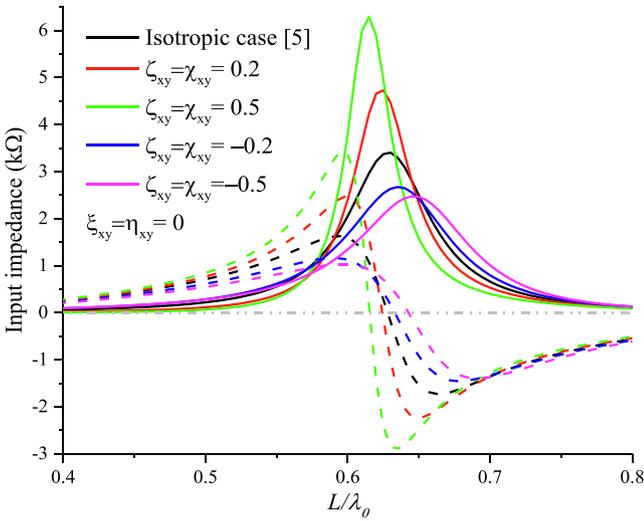
Both the maximum (peak) of the input impedance and the resonance frequency increase with the increasing positive values of  $\xi_{xy}$  and inversely for negative values. For  $\xi_{xy} = \eta_{xy} = 0.5$  ( $\xi_{xy} = \eta_{xy} = -0.5$ ) an increase (a decrease) of 50% of the input impedance peak is observed.

#### B. Effect of the reciprocal Tellegen medium ( $\chi_{xy} = \varsigma_{xy}$ ) on the input impedance and the resonance length

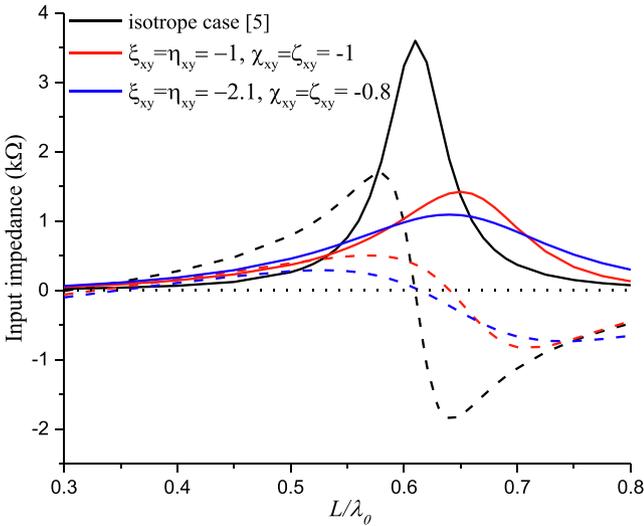
It should be noted that, in the case of positive values of  $\chi_{xy}$  (Fig. 3), the variation in the amplitude of the input impedance increases and conversely for negative values. On the other hand, the resonance points shift to the left of the isotropic case with respect to positive values. While negative values tend to move to the right as shown in Fig. 3, it is noticed that the Tellegen element exhibits a significant effect on the input impedance.

#### C. Combined effect of reciprocal chiral and reciprocal Tellegen elements on the input impedance and mutual coupling

The combined effect of chiral and Tellegen media, represented by complex valued magnetolectric elements (imaginary and real parts, respectively) on the input impedance and coupling is shown in Fig. 4. A selection of these elements values is made so that a reduced input impedance, for better matching, and an improved decoupling are obtained.



**Fig. 3** Effects of positive and negative reciprocal Tellegen bianisotropy on the input impedance of the printed dipole.



**Fig. 4** Input impedance for combined reciprocal chiral and Tellegen elements.

The first choice is based on the aforementioned case study ( $\chi_{xy} = \varsigma_{xy} = \xi_{xy} = \eta_{xy} = -1$ ). In this case, a large decrease in the input impedance peak with an increase in the resonant length are obtained. Another adjustment (second choice) is made to achieve a final peak decrease of more than 3 times, compared to the isotropic case, to reach  $1.02\text{k}\Omega$  for  $\chi_{xy} = \varsigma_{xy} = -0.8$  and  $\xi_{xy} = \eta_{xy} = -2.1$ , all without influencing the resonant frequency.

## 5. Conclusions

In this paper, the medium gyro-bianisotropy effect on the input impedance of the single dipole configuration is evaluated for reciprocal chiral, Tellegen and complex general gyro-reciprocal media. A selection of the magnetoelectric elements leads to a significant decrease in the input impedance, with

an increasing and decreasing of the dipole resonant length, which is advantageous for matching purposes, without modifying the resonance frequency compared to the isotropic case. Indeed, our results have shown that general complex media give more adequate parameters (4 parameters) to better control antenna parameters such as the input impedance. In addition, this paper opens perspectives on media with non-reciprocal gyrotropic anisotropy that contributes in the electromagnetic field expressions by a factor of  $e^{jk_0 \frac{(\varsigma_{xy} - \chi_{xy})}{2} z}$  or  $e^{jk_0 \frac{(\xi_{xy} - \eta_{xy})}{2} z}$  expressing phase or gain/loss, respectively, which must be deeply examined in future works.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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