Event-triggered recursive state estimation for dynamical networks under randomly switching topologies and multiple missing measurements ⋆

Jun Hu [a,b,∗], Zidong Wang [c], Guo-Ping Liu [a,d], Chaoqing Jia [b,e], Jonathan Williams [a]

[a] *School of Engineering, University of South Wales, Pontypridd CF37 1DL, UK*

[b] *Department of Mathematics, Harbin University of Science and Technology, Harbin 150080, China*

[c] *Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, UK*

[d] *Department of Artificial Intelligence and Automation, Wuhan University, Wuhan 430072, China*

[e] *Heilongjiang Provincial Key Laboratory of Optimization Control and Intelligent Analysis for Complex Systems, Harbin University of Science and Technology, Harbin 150080, China*

Abstract

In this paper, the design problem of recursive state estimator is discussed for a class of coupled nonlinear dynamical networks with randomly switching topologies and multiple missing measurements under the event-triggered mechanism. A sequence of random variables obeying the Bernoulli distribution with certain occurrence probabilities is adopted to model the multiple missing measurements and the random change manners of the network topologies. The event-based communication protocol is introduced to adjust the transmission frequency, thereby improving the energy utilization efficiencies of the communication networks. The objective of the addressed variance-constrained estimation problem is to construct a recursive state estimator such that, in the simultaneous presence of event-based transmission strategy, randomly switching topologies as well as multiple missing measurements, a locally optimal upper bound is guaranteed on the estimation error covariance by properly determining the estimator gain, where the desired estimator gain matrix is formulated via the solutions to certain recursive matrix equations. Besides, theoretical analysis is conducted on the monotonicity regarding the missing probabilities of degraded measurements and the obtained upper bound matrix. Finally, some simulations with comparisons are carried out to demonstrate the effectiveness and feasibility of proposed event-triggered state estimation method.

Key words: Recursive state estimation; Time-varying dynamical networks; Randomly switching topologies; Multiple missing measurements; Event-based communication mechanism; Monotonicity analysis.

1 Introduction

During the past few decades, particular research attention has been paid to the dynamics analysis issues of complex networks due to their widespread applications in a variety of domains such as sensor networks, social networks, electric power grids and so on [9, 15, 40]. So far, a number of efficient methods have been available to address the stability, synchronization, consensus and estimation problems for complex dynamical networks [3, 19, 32, 43]. Accordingly, several effective estimation strategies under different requirements/constraints have been developed to provide the desired estimation for the node states of complex dynamical networks [1, 5, 20]. For example, the finite-horizon state estimation scheme with satisfactory H_∞ performance has been given in [32] for time-varying uncertain dynamical networks by employing the recursive matrix inequality method, where the effects from nonlinear disturbances and degraded measurements have been examined simultaneously.

Owing to the possibly harsh network environments and severe interferences from other external factors, it is of vital significance to discuss the dynamical topologies structures when analyzing the behaviors of the complex networks [7, 25, 30, 37]. In fact, the issue of dynamically changing topology has recently received some initial research interest from the complex network community. For instance, in [25], the H_∞ state estimation scheme has been proposed for a class of nonlinear time-invariant dynamical networks subject to uncertain coupling strength and incomplete observations. In addition, the non-fragile H_∞ quantized estimation method has been developed in [37] for nonlinear dynamical networks subject to switching topologies and missing measurements, where the switching manner of the communication topology in the coupled configuration matrix has been characterized by the Markov chain. Regarding the time-varying case, in [19], the random coupling strength depicted by some variables obeying the uniform distri-

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∗ Corresponding author.

Email addresses: `hujun2013@gmail.com` (Jun Hu), `Zidong.Wang@brunel.ac.uk` (Zidong Wang).

bution has been examined and a recursive estimation approach of time-varying feature has been proposed. Very recently, a new state estimation algorithm has been developed in [7] for nonlinear time-varying dynamical networks in the presence of randomly varying topologies characterized by the Markovian jumping parameters and Kronecker delta function, where both the upper-bound variance constraint and H_∞ performance requirement can be achieved simultaneously if some recursive matrix equalities are solvable. So far, it should be pointed out that few methods can be available to deal with the stochastic fluctuations induced by the dynamically changing topologies, which deserves further investigation in order to better understand and analyze the spatial evolution behaviours of addressed dynamical networks.

As is well known, in a networked infrastructure, the phenomenon of missing measurements is unavoidable and, if not adequately handled, would seriously degrade or even invalidate the corresponding estimation/control performance [11, 12, 39]. As such, quite a few methodologies have been proposed in the literature to tackle the missing/degraded measurements [29, 33]. When it comes to the state estimation, in order to improve the algorithm performance, it is imperative to compensate the effects from the phenomenon of missing measurements during the course of estimator design for complex dynamical systems. So far, a rich body of literature has been available with a focus on the investigation of missing-measurement-induced effects on the state estimation performance [2, 4, 28, 41]. To mention just a few, in [21], the distributed state estimation strategy has been developed for sensor networks subject to missing measurements and randomly varying nonlinearities. In [22], a robust state estimation algorithm has been proposed for two-dimensional delayed stochastic systems with missing and saturated measurements. Nevertheless, it should be noticed that the recursive state estimation problems have not yet gained adequate research attention for dynamical networks catering for the time-varying circumstances [7, 41]. Very recently, a new recursive method has been given in [10] to deal with the state estimation problem under the variance-constraint for time-varying nonlinear dynamical networks subject to missing measurements. In contrast to the existing results, we aim to extend the proposed recursive state estimation method to examine the impacts from multiple missing measurements, where each sensor could have individual missing probability, with hope to better reflect the engineering practice.

On another research frontier, it is worth noting that neither the transmission capacities of the communication channels nor the data processing capability of the devices/components are unlimited [8, 31, 42]. Consequently, the event-based communication mechanism has aroused initial yet prevailing research attention due to its advantage in reducing unnecessary data transmissions and saving limited network resources [34]. In general, an event-based communication protocol provides certain communication criterion by means of the pre-defined threshold so as to adjust the transmission frequencies [18]. So far, many event-triggered conditions have been utilized in the literature and much work has been done on the event-based state estimation problem, see e.g. [13, 23, 27]. For example, an efficient steady-state Kalman estimation approach based on the event-triggered mechanism has been given in [18] for linear time-invariant stochastic systems subject to unknown inputs, where the increment of innovation vectors with pre-defined threshold has been employed to determine the transmission frequencies. In [13, 14], new remote state estimation algorithms have been developed for linear systems under stochastic event-triggered conditions. Besides, efficient optimal estimation approaches under two-point event-triggered scheme have been presented in [24, 26] for linear stochastic systems subject to hybrid measurements. Based on the differences with regard to the absolute error of measurements, the event-triggered estimation strategy under different performance indices have been presented in [18, 36] for nonlinear dynamical systems. Recently, in [31], the event-triggered state estimation algorithm via the argumentation method has been presented for *time-invariant* nonlinear dynamical networks with stochastic disturbances and mixed time-delays. However, the event-triggered recursive state estimation problem under variance-constraint has not been thoroughly addressed for *time-varying* complex dynamical networks, not to mention the situation where the addressed dynamical networks are subject to randomly switching topologies and multiple missing measurements simultaneously.

In light of the above discussions, in this paper, we endeavor to develop an event-based state estimation approach for coupled nonlinear dynamical networks in the presence of randomly switching topologies and multiple missing measurements. Here, we make one of the first few attempts to examine how the multiple missing measurements, randomly switching topologies as well as event-triggered protocol affect the overall estimation performance. Compared with the existing methods, the major difficulties encountered are: i) How to better understand and appropriately characterize the behaviours of the randomly switching topologies? ii) How to reflect the impacts caused by randomly switching topologies, multiple missing measurements and event-triggered communication strategy comprehensively and present an efficient estimation scheme accordingly? iii) How to show the decreasing characteristic of the estimation accuracy regarding the degraded measurements from the theoretical viewpoint? Hence, our main objective is to design a new state estimation scheme that recursively computes a certain locally optimized upper bound on the estimation error covariance and then analytically provide the estimator gain by solving some matrix difference equations. The major contributions can be summarized as follows: 1) the design problem of event-triggered state estimator under variance constraint is addressed, for the first time, for a class of nonlinear time-varying stochastic dynamical networks subject to randomly switching topologies and multiple missing measurements; 2) the expression of the desired time-varying estimator gain is characterized by seeking a certain upper bound of the error covariance and minimizing such an upper bound recursively; 3) the rigorously theoretical analysis is conducted with regard to the monotonicity of proposed estimation method; and 4) a new event-triggered estimation algorithm is proposed and outlined, which has a recursive way applicable for online implementations. Finally, some comparative simulations are carried out to demonstrate the validity of new event-based estimation approach presented in the paper.

Notations. The notations are generally standard in this paper. The set of $n \times m$ matrices is described by $\mathbb{R}^{n\times m}$. The n-dimensional Euclidean space is denoted by $\mathbb{R}^n$. Ω^T and Ω^{-1} represent the transpose and the inverse of Ω, respectively. $U > 0$ means that the matrix U is symmetric and positive-definite. I is an identity matrix of appropriate dimension. $\circ$ stands for the Hadamard product. $\mathbb{E}\{y\}$ is the expectation of the random variable y. Besides, if the dimensions of involved matrices are unknown, these matrices are supposed to be compatible for algebraic calculations.

2 Statement of The Problem

Consider the following class of discrete-time nonlinear coupled dynamical networks with randomly switching topologies and multiple missing measurements:

$$\begin{aligned} x_{i,k+1} =& f(x_{i,k}) + \alpha_{i,k}\sum_{j=1}^{N} w_{ij}^{(1)}(\Gamma + \xi_{i,k}\bar{\Gamma})x_{j,k} \\ &+(1-\alpha_{i,k})\sum_{j=1}^{N} w_{ij}^{(2)}(\Gamma + \xi_{i,k}\bar{\Gamma})x_{j,k} \\ &+B_{i,k}\varpi_{i,k}, \qquad (1)\\ y_{i,k} =& \Pi_{i,k}C_{i,k}x_{i,k} + \nu_{i,k}, \qquad (2)\end{aligned}$$

where $x_{i,k} \in \mathbb{R}^n$ represents the system state of the i-th node to be estimated with initial value $x_{i,0}$, $y_{i,k} \in \mathbb{R}^m$ stands for the measurement output of the i-th node, $W^{(1)} = \left[\, w_{ij}^{(1)} \,\right]_{N\times N}$ and $W^{(2)} = \left[\, w_{ij}^{(2)} \,\right]_{N\times N}$ are the coupling strength matrices with $w_{ij}^{(1)}$ and $w_{ij}^{(2)}$ $(i \neq j)$ being not all zero, and $\Gamma = \mathrm{diag}\{r_1, r_2, \cdots, r_n\}$ is a known inner-coupling matrix. $f(x_{i,k})$ is a continuously differentiable nonlinear function. $\xi_{i,k} \in \mathbb{R}$ is a zero-mean white noise sequence with variance $\mathbb{E}\{\xi_{i,k}^2\} = 1$, $\varpi_{i,k} \in \mathbb{R}^p$ represents the process noise with mean 0 and covariance $W_{i,k}$. $\nu_{i,k} \in \mathbb{R}^m$ denotes the zero-mean measurement noise with covariance $V_{i,k} > 0$. $\Pi_{i,k} = \mathrm{diag}\{\pi_{i,k}^{(1)}, \pi_{i,k}^{(2)}, \cdots, \pi_{i,k}^{(m)}\}$ with $\pi_{i,k}^{(l)}$ $(l = 1, 2, \cdots, m)$ being the Bernoulli distributed random variables. $B_{i,k}$, $\bar{\Gamma}$ and $C_{i,k}$ are known coefficient matrices.

The phenomena of randomly switching topologies and multiple missing measurements are modelled by Bernoulli distributed variables $\alpha_{i,k} \in \mathbb{R}$ and $\pi_{i,k}^{(l)} \in \mathbb{R}$ satisfying

$$\mathrm{Prob}\{\alpha_{i,k} = 1\} = \mathbb{E}\{\alpha_{i,k}\} = \bar{\alpha}_{i,k}, \qquad (3)$$

$$\mathrm{Prob}\{\pi_{i,k}^{(l)} = 1\} = \mathbb{E}\{\pi_{i,k}^{(l)}\} = \bar{\pi}_{i,k}^{(l)}, \qquad (4)$$

where $\bar{\alpha}_{i,k} \in [0,1]$ and $\bar{\pi}_{i,k}^{(l)} \in [0,1]$ are known constants. In what follows, suppose that $\alpha_{i,k}$, $\varpi_{i,k}$, $\xi_{i,k}$ $\nu_{i,k}$, $x_{i,0}$ and $\pi_{i,k}^{(l)}$ $(l = 1, 2, \cdots, m)$ are mutually independent.

Remark 1 *In (1), the stochastic inner-coupling phenomenon is considered, where the matrices Γ, $\bar{\Gamma}$ and the white noise $\xi_{i,k}$ are employed to model this phenomenon. In particular, the scalars r_i $(i = 1, 2, \cdots, N)$ stand for the inner coupling strength of node states. It is worthwhile to mention that the coupling strengths among the node elements could be different if $r_i \neq r_j$. Moreover, the stochastic inner-coupling modelling errors are characterized by utilizing $\bar{\Gamma}$ and $\xi_{i,k}$, which could raise more flexibility on the mathematical modelling of dynamical networks.*

In the sequel, the event-based communication mechanism is employed to save limited network resources and reduce the power consumption. Specifically, the following signal transmission strategy is employed:

$$(y_{i,k+s} - y_{i,k_m})^T(y_{i,k+s} - y_{i,k_m}) > \delta_i, \qquad (5)$$

where $\delta_i > 0$ is the triggered threshold, $y_{i,k+s}$ and y_{i,k_m} represent the current measurement output and the latest transmitted measurement, respectively. For the event-triggered time k_m with $k_0 = 0$, the transmitted signal can be denoted as follows:

$$\bar{y}_{i,k} = y_{i,k_m}, \quad k \in \{k_m, k_m + 1, \cdots, k_{m+1} - 1\}.$$

For the i-th node, the following recursive state estimator based on the available measurements is constructed:

$$\begin{aligned}\hat{x}_{i,k+1|k} =& f(\hat{x}_{i,k|k}) + \bar{\alpha}_{i,k}\sum_{j=1}^{N} w_{ij}^{(1)}\Gamma\hat{x}_{j,k|k} \\ &+(1-\bar{\alpha}_{i,k})\sum_{j=1}^{N} w_{ij}^{(2)}\Gamma\hat{x}_{j,k|k}, \qquad (6)\\ \hat{x}_{i,k+1|k+1} =& \hat{x}_{i,k+1|k} + \mathcal{K}_{i,k+1}(\bar{y}_{i,k+1} - \bar{\Pi}_{i,k+1} \\ &\times C_{i,k+1}\hat{x}_{i,k+1|k}), \qquad (7)\end{aligned}$$

where $\hat{x}_{i,k+1|k}$ and $\hat{x}_{i,k+1|k+1}$ stand for the prediction estimation and state estimation of $x_{i,k}$ respectively, $\bar{\Pi}_{i,k+1} = \mathbb{E}\{\Pi_{i,k+1}\} = \mathrm{diag}\{\bar{\pi}_{i,k}^{(1)}, \bar{\pi}_{i,k}^{(2)}, \cdots, \bar{\pi}_{i,k}^{(m)}\}$ with $\bar{\pi}_{i,k}^{(l)}$ being the mathematical expectation of $\pi_{i,k}^{(l)}$ $(l = 1, 2, \cdots, m)$, and $\mathcal{K}_{i,k+1}$ represents the desired state estimator parameter matrix to be determined.

Let the prediction error and estimation error be denoted by $e_{i,k+1|k} = x_{i,k+1} - \hat{x}_{i,k+1|k}$ and $e_{i,k+1|k+1} = x_{i,k+1} - \hat{x}_{i,k+1|k+1}$, respectively. Moreover, let the prediction error covariance matrix be $X_{i,k+1|k} = \mathbb{E}\{e_{i,k+1|k}e_{i,k+1|k}^T\}$ and the estimation error covariance matrix be $X_{i,k+1|k+1} = \mathbb{E}\{e_{i,k+1|k+1}e_{i,k+1|k+1}^T\}$, respectively. In the following, we aim to construct a variance-constrained estimator of recursive form (6) and (7) such that: 1) there exists an upper bound covariance matrix $\mathcal{X}_{i,k+1|k+1}$ of $X_{i,k+1|k+1}$; and 2) such an upper bound covariance matrix $\mathcal{X}_{i,k+1|k+1}$ is minimized by choosing the estimator parameter matrix $\mathcal{K}_{i,k+1}$ properly. In addition, we aim to illustrate the monotonicity regarding the occurrence probabilities of missing measurements as well as the optimized upper bound.

Remark 2 *The time-varying estimator (6) and (7) possesses the following features and advantages: a) the de-*

signed state estimator includes the prediction and innovation updating steps, which has certain error correction ability; b) the dimension of new state estimator is consistent with the one of original node state; c) the constructed state estimator is in a distributed way without utilizing the state augmentation method, thereby avoiding the issue of computational burdens; d) the available information of randomly switching topologies and multiple missing measurements is employed during the estimator design, hence those induced effects will be well examined in the presented estimation algorithm; and e) the time-varying characteristic is well reflected in (6) and (7), therefore the new recursive estimation scheme designed later is applicable for online implementations.

3 Design of Estimation Algorithm

In this section, the covariance matrices of prediction error and estimation error are firstly calculated based on corresponding definitions. Next, the recursion equation of an optimal upper bound on the estimation error covariance matrix is established and the state estimator gain is appropriately determined to optimize the trace of $\mathcal{X}_{i,k+1|k+1}$.

To begin with, we calculate the prediction error $e_{i,k+1|k}$. Subtracting (6) from (1) leads to

$$\begin{aligned} e_{i,k+1|k} = & \sum_{j=1}^{N}\left[\bar{\alpha}_{i,k}\omega_{ij}^{(1)} + (1-\bar{\alpha}_{i,k})\omega_{ij}^{(2)}\right]\Gamma e_{j,k|k} \\ & +\tilde{\alpha}_{i,k}\sum_{j=1}^{N}\left(\omega_{ij}^{(1)}-\omega_{ij}^{(2)}\right)\Gamma x_{j,k} \\ & +\sum_{j=1}^{N}\left[\alpha_{i,k}\omega_{ij}^{(1)} + (1-\alpha_{i,k})\omega_{ij}^{(2)}\right]\xi_{i,k}\bar{\Gamma}x_{j,k} \\ & +f(x_{i,k}) - f(\hat{x}_{i,k|k}) + B_{i,k}\varpi_{i,k}, \end{aligned} \tag{8}$$

where $\tilde{\alpha}_{i,k} = \alpha_{i,k} - \bar{\alpha}_{i,k}$. For the nonlinearity $f(x_{i,k})$, taking the Taylor series expansion around $\hat{x}_{i,k|k}$ as in [38], we obtain

$$f(x_{i,k}) = f(\hat{x}_{i,k|k}) + A_{i,k}e_{i,k|k} + o(|e_{i,k|k}|), \tag{9}$$

where $A_{i,k} = \partial f(x_{i,k})/\partial x_{i,k}|_{x_{i,k}=\hat{x}_{i,k|k}}$. It follows from [38] that the resultant high-order term is approximately estimated, which takes the following form

$$o(|e_{i,k|k}|) = F_{i,k}\Im_{i,k}L_{i,k}e_{i,k|k}, \tag{10}$$

where $L_{i,k}$ is utilized to adjust the freedom degree so as to tune the state estimator, $F_{i,k}$ is a known problem-dependent matrix, and the unknown time-varying matrix $\Im_{i,k}$ is adopted to describe the linearization errors satisfying $\Im_{i,k}\Im_{i,k}^T \leq I$.

Substituting (9) and (10) into (8), we know that

$$\begin{aligned} e_{i,k+1|k} = & \sum_{j=1}^{N}\left[\bar{\alpha}_{i,k}\omega_{ij}^{(1)} + (1-\bar{\alpha}_{i,k})\omega_{ij}^{(2)}\right]\Gamma e_{j,k|k} \\ & +\tilde{\alpha}_{i,k}\sum_{j=1}^{N}\left(\omega_{ij}^{(1)}-\omega_{ij}^{(2)}\right)\Gamma x_{j,k} \\ & +\sum_{j=1}^{N}\left[\alpha_{i,k}\omega_{ij}^{(1)} + (1-\alpha_{i,k})\omega_{ij}^{(2)}\right]\xi_{i,k}\bar{\Gamma}x_{j,k} \\ & +(A_{i,k} + F_{i,k}\Im_{i,k}L_{i,k})e_{i,k|k} + B_{i,k}\varpi_{i,k}. \end{aligned} \tag{11}$$

Next, the zero term $\mathcal{K}_{i,k+1}y_{i,k+1} - \mathcal{K}_{i,k+1}y_{i,k+1}$ is added to the right-hand side of equation (7) and then the estimation error can be described by:

$$\begin{aligned} e_{i,k+1|k+1} = & (I - \mathcal{K}_{i,k+1}\bar{\Pi}_{i,k+1}C_{i,k+1})e_{i,k+1|k} \\ & -\mathcal{K}_{i,k+1}\tilde{\Pi}_{i,k+1}C_{i,k+1}x_{i,k+1} \\ & -\mathcal{K}_{i,k+1}(\bar{y}_{i,k+1} - y_{i,k+1}) \\ & -\mathcal{K}_{i,k+1}\nu_{i,k+1}, \end{aligned} \tag{12}$$

where $\tilde{\Pi}_{i,k+1} = \Pi_{i,k+1} - \bar{\Pi}_{i,k+1}$. Here, it is easy to know that $\mathbb{E}\{\tilde{\Pi}_{i,k+1}\} = 0$.

To facilitate the subsequent theoretical developments, the following Lemmas are introduced.

Lemma 1 *[35] For given matrices A, H, E and M satisfying $MM^T \leq I$, if $U > 0$ and $\tau > 0$ satisfy $\tau^{-1}I - EUE^T > 0$, then*

$$\begin{aligned} & (A + HME)U(A + HME)^T \\ \leq & A(U^{-1} - \tau E^T E)^{-1}A^T + \tau^{-1}HH^T. \end{aligned}$$

Lemma 2 *[16] Let $A = [a_{ij}]_{n\times n}$ be a real-value matrix and $D = \mathrm{diag}\{d_1, d_2, \cdots, d_n\}$ denotes a stochastic matrix. Then, the following equation holds:*

$$\mathbb{E}\{DAD^T\} = \begin{bmatrix} \mathbb{E}\{d_1^2\} & \mathbb{E}\{d_1 d_2\} & \cdots & \mathbb{E}\{d_1 d_n\} \\ \mathbb{E}\{d_2 d_1\} & \mathbb{E}\{d_2^2\} & \cdots & \mathbb{E}\{d_2 d_n\} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{E}\{d_n d_1\} & \mathbb{E}\{d_n d_2\} & \cdots & \mathbb{E}\{d_n^2\} \end{bmatrix} \circ A,$$

where $\circ$ represents the Hadamard product with $[J \circ L]_{ij} = J_{ij} \cdot L_{ij}$.

Now, based on the definitions of the covariance matrices of prediction error and state estimation error, we aim to obtain the recursion equation of upper bound on the estimation error covariance matrix.

Theorem 1 *Consider the nonlinear time-varying coupled complex dynamical networks (1)-(2) with the recursive estimator given by (6)-(7). The prediction error covariance matrix $X_{i,k+1|k}$ is described by:*

$$\begin{aligned} & X_{i,k+1|k} \\ = & \Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\vec{\omega}_{ij}\vec{\omega}_{is}\mathbb{E}\{e_{j,k|k}e_{s,k|k}^T\}\Gamma^T \\ & +\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\bar{\alpha}_{i,k}(1-\bar{\alpha}_{i,k})\ddot{\omega}_{ij}\ddot{\omega}_{is}\mathbb{E}\{x_{j,k}x_{s,k}^T\}\Gamma^T \end{aligned}$$

$$
\begin{aligned}
&+\bar{\Gamma}\sum_{j=1}^{N}\sum_{s=1}^{N}\bar{\alpha}_{i,k}\omega_{ij}^{(1)}\omega_{is}^{(1)}\mathbb{E}\{x_{j,k}x_{s,k}^T\}\bar{\Gamma}^T\\
&+\bar{\Gamma}\sum_{j=1}^{N}\sum_{s=1}^{N}(1-\bar{\alpha}_{i,k})\omega_{ij}^{(2)}\omega_{is}^{(2)}\mathbb{E}\{x_{j,k}x_{s,k}^T\}\bar{\Gamma}^T\\
&+(A_{i,k}+F_{i,k}\Im_{i,k}L_{i,k})X_{i,k|k}(A_{i,k}+F_{i,k}\Im_{i,k}L_{i,k})^T\\
&+B_{i,k}W_{i,k}B_{i,k}^T+\mathfrak{N}_1+\mathfrak{N}_1^T, \qquad (13)
\end{aligned}
$$

where

$$
\vec{\omega}_{ij}=\bar{\alpha}_{i,k}\omega_{ij}^{(1)}+(1-\bar{\alpha}_{i,k})\omega_{ij}^{(2)},\ \ \ddot{\omega}_{ij}=\omega_{ij}^{(1)}-\omega_{ij}^{(2)},
$$

$$
\mathfrak{N}_1=\Gamma\sum_{j=1}^{N}\vec{\omega}_{ij}\mathbb{E}\{e_{j,k|k}e_{i,k|k}^T\}(A_{i,k}+F_{i,k}\Im_{i,k}L_{i,k})^T.
$$

Proof: In terms of the covariance definition of prediction error, we have

$$
\begin{aligned}
&X_{i,k+1|k}\\
&=\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\vec{\omega}_{ij}\vec{\omega}_{is}\mathbb{E}\{e_{j,k|k}e_{s,k|k}^T\}\Gamma^T\\
&+\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\bar{\alpha}_{i,k}(1-\bar{\alpha}_{i,k})\ddot{\omega}_{ij}\ddot{\omega}_{is}\mathbb{E}\{x_{j,k}x_{s,k}^T\}\Gamma^T\\
&+\bar{\Gamma}\sum_{j=1}^{N}\sum_{s=1}^{N}\bar{\alpha}_{i,k}\omega_{ij}^{(1)}\omega_{is}^{(1)}\mathbb{E}\{x_{j,k}x_{s,k}^T\}\bar{\Gamma}^T\\
&+\bar{\Gamma}\sum_{j=1}^{N}\sum_{s=1}^{N}(1-\bar{\alpha}_{i,k})\omega_{ij}^{(2)}\omega_{is}^{(2)}\mathbb{E}\{x_{j,k}x_{s,k}^T\}\bar{\Gamma}^T\\
&+(A_{i,k}+F_{i,k}\Im_{i,k}L_{i,k})X_{i,k|k}(A_{i,k}+F_{i,k}\Im_{i,k}L_{i,k})^T\\
&+B_{i,k}W_{i,k}B_{i,k}^T+\sum_{r=1}^{10}\left(\mathfrak{N}_r+\mathfrak{N}_r^T\right),
\end{aligned}
$$

where

$$
\begin{aligned}
\mathfrak{N}_2&=\mathbb{E}\left\{\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\vec{\omega}_{ij}\ddot{\omega}_{is}\tilde{\alpha}_{i,k}e_{j,k|k}x_{s,k}^T\Gamma^T\right\},\\
\mathfrak{N}_3&=\mathbb{E}\left\{\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\vec{\omega}_{ij}[\alpha_{i,k}\omega_{is}^{(1)}+(1-\alpha_{i,k})\omega_{is}^{(2)}]\right.\\
&\qquad\left.\times\xi_{i,k}e_{j,k|k}x_{s,k}^T\bar{\Gamma}^T\right\},\\
\mathfrak{N}_4&=\mathbb{E}\left\{\Gamma\sum_{j=1}^{N}\vec{\omega}_{ij}e_{j,k|k}\varpi_{i,k}^TB_{i,k}^T\right\},\\
\mathfrak{N}_5&=\mathbb{E}\left\{\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\ddot{\omega}_{ij}[\alpha_{i,k}\omega_{is}^{(1)}+(1-\alpha_{i,k})\omega_{is}^{(2)}]\right.\\
&\qquad\left.\times\tilde{\alpha}_{i,k}\xi_{i,k}x_{j,k}x_{s,k}^T\bar{\Gamma}^T\right\},\\
\mathfrak{N}_6&=\mathbb{E}\left\{\Gamma\sum_{j=1}^{N}\ddot{\omega}_{ij}\tilde{\alpha}_{i,k}x_{j,k}e_{i,k|k}^T(A_{i,k}+F_{i,k}\Im_{i,k}L_{i,k})^T\right\},\\
\mathfrak{N}_7&=\mathbb{E}\left\{\Gamma\sum_{j=1}^{N}\ddot{\omega}_{ij}\tilde{\alpha}_{i,k}x_{j,k}\varpi_{i,k}^TB_{i,k}^T\right\},\\
\mathfrak{N}_8&=\mathbb{E}\left\{\bar{\Gamma}\sum_{j=1}^{N}[\alpha_{i,k}\omega_{ij}^{(1)}+(1-\alpha_{i,k})\omega_{ij}^{(2)}]\xi_{i,k}x_{j,k}e_{i,k|k}^T\right.\\
&\qquad\left.\times(A_{i,k}+F_{i,k}\Im_{i,k}L_{i,k})^T\right\},\\
\mathfrak{N}_9&=\mathbb{E}\left\{\bar{\Gamma}\sum_{j=1}^{N}[\alpha_{i,k}\omega_{ij}^{(1)}+(1-\alpha_{i,k})\omega_{ij}^{(2)}]\xi_{i,k}x_{j,k}\varpi_{i,k}^TB_{i,k}^T\right\},\\
\mathfrak{N}_{10}&=(A_{i,k}+F_{i,k}\Im_{i,k}L_{i,k})\mathbb{E}\{e_{i,k|k}\varpi_{i,k}^T\}B_{i,k}^T.
\end{aligned}
$$

Recalling the independence property of all involved random variables and together with $\mathbb{E}\{\tilde{\alpha}_{i,k}\}=0$, $\mathbb{E}\{\xi_{i,k}\}=0$ and $\mathbb{E}\{\varpi_{i,k}\}=0$, it is easy to see that $\mathfrak{N}_i=0$ ($i=2,\cdots,10$) and the proof is then complete.

Theorem 2 *Consider the nonlinear coupled dynamical networks (1)-(2) with the recursive estimator given by (6)-(7). The recursive equation of estimation error covariance matrix $X_{i,k+1|k+1}$ is described as follows:*

$$
\begin{aligned}
&X_{i,k+1|k+1}\\
&=(I-\mathcal{K}_{i,k+1}\bar{\Pi}_{i,k+1}C_{i,k+1})X_{i,k+1|k}(I-\mathcal{K}_{i,k+1}\\
&\times\bar{\Pi}_{i,k+1}C_{i,k+1})^T+\mathcal{K}_{i,k+1}\mathbb{E}\{(\bar{y}_{i,k+1}-y_{i,k+1})\\
&\times(\bar{y}_{i,k+1}-y_{i,k+1})^T\}\mathcal{K}_{i,k+1}^T+\mathcal{K}_{i,k+1}\mathbb{E}\{\tilde{\Pi}_{i,k+1}C_{i,k+1}\\
&\times x_{i,k+1}x_{i,k+1}^TC_{i,k+1}^T\tilde{\Pi}_{i,k+1}\}\mathcal{K}_{i,k+1}^T\\
&+\mathcal{K}_{i,k+1}V_{i,k+1}\mathcal{K}_{i,k+1}^T-\mathfrak{M}_1-\mathfrak{M}_1^T\\
&+\mathfrak{M}_2+\mathfrak{M}_2^T+\mathfrak{M}_3+\mathfrak{M}_3^T, \qquad (14)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathfrak{M}_1&=(I-\mathcal{K}_{i,k+1}\bar{\Pi}_{i,k+1}C_{i,k+1})\mathbb{E}\{e_{i,k+1|k}\\
&\qquad\times(\bar{y}_{i,k+1}-y_{i,k+1})^T\}\mathcal{K}_{i,k+1}^T,\\
\mathfrak{M}_2&=\mathcal{K}_{i,k+1}\mathbb{E}\{(\bar{y}_{i,k+1}-y_{i,k+1})x_{i,k+1}^TC_{i,k+1}^T\tilde{\Pi}_{i,k+1}\}\mathcal{K}_{i,k+1}^T,\\
\mathfrak{M}_3&=\mathcal{K}_{i,k+1}\mathbb{E}\{(\bar{y}_{i,k+1}-y_{i,k+1})\nu_{i,k+1}^T\}\mathcal{K}_{i,k+1}^T.
\end{aligned}
$$

Proof: According to the covariance definition of estimation error, we obtain

$$
\begin{aligned}
&X_{i,k+1|k+1}\\
&=(I-\mathcal{K}_{i,k+1}\bar{\Pi}_{i,k+1}C_{i,k+1})X_{i,k+1|k}(I-\mathcal{K}_{i,k+1}\\
&\times\bar{\Pi}_{i,k+1}C_{i,k+1})^T+\mathcal{K}_{i,k+1}\mathbb{E}\{(\bar{y}_{i,k+1}-y_{i,k+1})\\
&\times(\bar{y}_{i,k+1}-y_{i,k+1})^T\}\mathcal{K}_{i,k+1}^T+\mathcal{K}_{i,k+1}\mathbb{E}\{\tilde{\Pi}_{i,k+1}C_{i,k+1}\\
&\times x_{i,k+1}x_{i,k+1}^TC_{i,k+1}^T\tilde{\Pi}_{i,k+1}\}\mathcal{K}_{i,k+1}^T\\
&+\mathcal{K}_{i,k+1}V_{i,k+1}\mathcal{K}_{i,k+1}^T-\mathfrak{M}_1-\mathfrak{M}_1^T+\mathfrak{M}_2+\mathfrak{M}_2^T\\
&+\mathfrak{M}_3+\mathfrak{M}_3^T-\mathfrak{M}_4-\mathfrak{M}_4^T-\mathfrak{M}_5-\mathfrak{M}_5^T+\mathfrak{M}_6+\mathfrak{M}_6^T,
\end{aligned}
$$

where

$$\begin{aligned}\mathfrak{M}_4 =& (I-\mathcal{K}_{i,k+1}\bar{\Pi}_{i,k+1}C_{i,k+1})\mathbb{E}\{e_{i,k+1|k}x_{i,k+1}^T\\ &\times C_{i,k+1}^T\tilde{\Pi}_{i,k+1}\}\mathcal{K}_{i,k+1}^T,\\ \mathfrak{M}_5 =& (I-\mathcal{K}_{i,k+1}\bar{\Pi}_{i,k+1}C_{i,k+1})\mathbb{E}\{e_{i,k+1|k}\nu_{i,k+1}^T\}\mathcal{K}_{i,k+1}^T,\\ \mathfrak{M}_6 =& \mathcal{K}_{i,k+1}\mathbb{E}\{\tilde{\Pi}_{i,k+1}C_{i,k+1}x_{i,k+1}\nu_{i,k+1}^T\}\mathcal{K}_{i,k+1}^T.\end{aligned}$$

Similarly, it is readily known that $\mathfrak{M}_t = 0$ $(t = 4,5,6)$ and (14) holds. Then, the proof is complete.

Remark 3 *So far, the covariance matrices of prediction error and estimation error are given in Theorem 1 and Theorem 2, respectively. Generally, it is expected to minimize the estimation error covariance and design the estimator parameter matrix $\mathcal{K}_{i,k+1}$ simultaneously, thereby presenting a globally optimal solution. Unfortunately, it is extremely difficult to obtain the accurate values of $X_{i,k+1|k}$ and $X_{i,k+1|k+1}$ because of some unknown terms in (13)-(14), which are induced by the random changes of the network topologies, the event-based communication protocol and the linearization errors. In the sequel, a local optimization approach is proposed to obtain the upper bound on the estimation error covariance and the recursive form of the state estimator parameter. Hence, a sub-optimal estimation scheme is obtained to retain admissible estimation performance.*

Theorem 3 *Consider the recursion equations of the prediction error covariance matrix in (13) and the estimation error covariance matrix in (14). For $\varrho_t > 0$ $(t = 1,2,\cdots,8)$ and $\gamma_{i,k} > 0$, suppose that the following two recursive matrix difference equations*

$$\begin{aligned}&\mathcal{X}_{i,k+1|k}\\ =& \sum_{j=1}^{N}\omega_{ij}^{'}\Gamma\mathcal{X}_{j,k|k}\Gamma^T + \ddot{\omega}_i\bar{\alpha}_{i,k}(1-\bar{\alpha}_{i,k})(1+\varrho_2^{-1})\\ &\times\sum_{j=1}^{N}\ddot{\omega}_{ij}\Gamma\hat{x}_{j,k|k}\hat{x}_{j,k|k}^T\Gamma^T + \sum_{j=1}^{N}\omega_{ij}^{''}\bar{\Gamma}\mathcal{X}_{j,k|k}\bar{\Gamma}^T\\ &+\sum_{j=1}^{N}\omega_{ij}^{'''}\bar{\Gamma}\hat{x}_{j,k|k}\hat{x}_{j,k|k}^T\bar{\Gamma}^T + B_{i,k}W_{i,k}B_{i,k}^T\\ &+(1+\varrho_1^{-1})\Big[A_{i,k}(\mathcal{X}_{i,k|k}^{-1}-\gamma_{i,k}L_{i,k}^TL_{i,k})^{-1}A_{i,k}^T\\ &+\gamma_{i,k}^{-1}F_{i,k}F_{i,k}^T\Big] \qquad (15)\end{aligned}$$

and

$$\begin{aligned}&\mathcal{X}_{i,k+1|k+1}\\ =& (1+\varrho_5)(I-\mathcal{K}_{i,k+1}\bar{\Pi}_{i,k+1}C_{i,k+1})\mathcal{X}_{i,k+1|k}\\ &\times(I-\mathcal{K}_{i,k+1}\bar{\Pi}_{i,k+1}C_{i,k+1})^T + \mathcal{K}_{i,k+1}\\ &\times\Big\{(1+\varrho_5^{-1}+\varrho_6+\varrho_7)\delta_iI + (1+\varrho_7^{-1})V_{i,k+1}\\ &+(1+\varrho_6^{-1})\Xi_{i,k+1}\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T)\Big\}\mathcal{K}_{i,k+1}^T \qquad (16)\end{aligned}$$

under the following inequality constraint

$$\gamma_{i,k}^{-1}I - L_{i,k}\mathcal{X}_{i,k|k}L_{i,k}^T > 0 \qquad (17)$$

have the solutions $\mathcal{X}_{i,k+1|k} > 0$ and $\mathcal{X}_{i,k+1|k+1} > 0$, where

$$\begin{aligned}\omega_{ij}^{'} =& (1+\varrho_1)\vec{\omega}_i\vec{\omega}_{ij} + \ddot{\omega}_i\bar{\alpha}_{i,k}(1-\bar{\alpha}_{i,k})(1+\varrho_2)\ddot{\omega}_{ij},\\ \omega_{ij}^{''} =& \bar{\alpha}_{i,k}\omega_i^{(1)}(1+\varrho_3)\omega_{ij}^{(1)} + \omega_i^{(2)}(1-\bar{\alpha}_{i,k})(1+\varrho_4)\omega_{ij}^{(2)},\\ \omega_{ij}^{'''} =& \bar{\alpha}_{i,k}\omega_i^{(1)}(1+\varrho_3^{-1})\omega_{ij}^{(1)} + \omega_i^{(2)}(1-\bar{\alpha}_{i,k})\\ &\times(1+\varrho_4^{-1})\omega_{ij}^{(2)},\\ \vec{\omega}_i =& \sum_{j=1}^{N}\vec{\omega}_{ij},\ \ddot{\omega}_i = \sum_{j=1}^{N}\ddot{\omega}_{ij},\\ \omega_i^{(1)} =& \sum_{j=1}^{N}\omega_{ij}^{(1)},\ \omega_i^{(2)} = \sum_{j=1}^{N}\omega_{ij}^{(2)},\\ \bar{\Psi}_{i,k+1} =& (1+\varrho_8)\mathcal{X}_{i,k+1|k} + (1+\varrho_8^{-1})\hat{x}_{i,k+1|k}\hat{x}_{i,k+1|k}^T,\\ \Xi_{i,k+1} =& \mathrm{diag}\big\{\bar{\pi}_{i,k+1}^{(1)}(1-\bar{\pi}_{i,k+1}^{(1)}),\bar{\pi}_{i,k+1}^{(2)}(1-\bar{\pi}_{i,k+1}^{(2)}),\\ &\cdots,\bar{\pi}_{i,k+1}^{(m)}(1-\bar{\pi}_{i,k+1}^{(m)})\big\}. \qquad (18)\end{aligned}$$

Then, it can be shown that $\mathcal{X}_{i,k+1|k+1}$ is an upper bound of $X_{i,k+1|k+1}$, i.e.,

$$X_{i,k+1|k+1} \leq \mathcal{X}_{i,k+1|k+1}.$$

Furthermore, if the estimator parameter matrix $\mathcal{K}_{i,k+1}$ is characterized by

$$\begin{aligned}&\mathcal{K}_{i,k+1}\\ =& (1+\varrho_5)\mathcal{X}_{i,k+1|k}C_{i,k+1}^T\bar{\Pi}_{i,k+1}\Big\{(1+\varrho_5)\bar{\Pi}_{i,k+1}C_{i,k+1}\\ &\times\mathcal{X}_{i,k+1|k}C_{i,k+1}^T\bar{\Pi}_{i,k+1} + (1+\varrho_5^{-1}+\varrho_6+\varrho_7)\delta_iI\\ &+(1+\varrho_7^{-1})V_{i,k+1} + (1+\varrho_6^{-1})\Xi_{i,k+1}\\ &\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T)\Big\}^{-1}, \qquad (19)\end{aligned}$$

then it can be testified that $\mathrm{tr}\left(\mathcal{X}_{i,k+1|k+1}\right)$ is minimized at every sampling step.

Proof: This theorem is proved by utilizing the mathematical induction method and the proof process includes the following three steps.

Step 1: Let us deal with the cross and unknown terms in (13). In terms of the following preliminary inequality

$$xy^T + yx^T \leq \varrho xx^T + \varrho^{-1}yy^T \qquad (20)$$

with $x, y \in \mathbb{R}^n$ and $\varrho > 0$ being a scalar, one has

$$\begin{aligned}&\mathfrak{N}_1 + \mathfrak{N}_1^T\\ \leq& \varrho_1\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\vec{\omega}_{ij}\vec{\omega}_{is}\mathbb{E}\{e_{j,k|k}e_{s,k|k}^T\}\Gamma^T + \varrho_1^{-1}(A_{i,k}\\ &+F_{i,k}\Im_{i,k}L_{i,k})X_{i,k|k}(A_{i,k}+F_{i,k}\Im_{i,k}L_{i,k})^T \qquad (21)\end{aligned}$$

where $\varrho_1 > 0$. From (13) and (21), it follows that

$$X_{i,k+1|k}$$

$$\leq (1+\varrho_1)\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\vec{\omega}_{ij}\vec{\omega}_{is}\mathbb{E}\{e_{j,k|k}e_{s,k|k}^T\}\Gamma^T$$
$$+\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\bar{\alpha}_{i,k}(1-\bar{\alpha}_{i,k})\ddot{\omega}_{ij}\ddot{\omega}_{is}\mathbb{E}\{x_{j,k}x_{s,k}^T\}\Gamma^T$$
$$+\bar{\Gamma}\sum_{j=1}^{N}\sum_{s=1}^{N}\bar{\alpha}_{i,k}\omega_{ij}^{(1)}\omega_{is}^{(1)}\mathbb{E}\{x_{j,k}x_{s,k}^T\}\bar{\Gamma}^T$$
$$+\bar{\Gamma}\sum_{j=1}^{N}\sum_{s=1}^{N}(1-\bar{\alpha}_{i,k})\omega_{ij}^{(2)}\omega_{is}^{(2)}\mathbb{E}\{x_{j,k}x_{s,k}^T\}\bar{\Gamma}^T$$
$$+(1+\varrho_1^{-1})(A_{i,k}+F_{i,k}\Im_{i,k}L_{i,k})X_{i,k|k}$$
$$\times(A_{i,k}+F_{i,k}\Im_{i,k}L_{i,k})^T+B_{i,k}W_{i,k}B_{i,k}^T. \tag{22}$$

Recalling (20), we have

$$\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\vec{\omega}_{ij}\vec{\omega}_{is}\mathbb{E}\{e_{j,k|k}e_{s,k|k}^T\}\Gamma^T$$
$$\leq \frac{1}{2}\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\vec{\omega}_{ij}\vec{\omega}_{is}\mathbb{E}\{e_{j,k|k}e_{j,k|k}^T+e_{s,k|k}e_{s,k|k}^T\}\Gamma^T$$
$$=\frac{1}{2}\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\vec{\omega}_{ij}\vec{\omega}_{is}\mathbb{E}\{e_{j,k|k}e_{j,k|k}^T\}\Gamma^T$$
$$+\frac{1}{2}\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\vec{\omega}_{ij}\vec{\omega}_{is}\mathbb{E}\{e_{s,k|k}e_{s,k|k}^T\}\Gamma^T$$
$$=\frac{1}{2}\Gamma\sum_{s=1}^{N}\vec{\omega}_{is}\sum_{j=1}^{N}\vec{\omega}_{ij}\mathbb{E}\{e_{j,k|k}e_{j,k|k}^T\}\Gamma^T$$
$$+\frac{1}{2}\Gamma\sum_{j=1}^{N}\vec{\omega}_{ij}\sum_{s=1}^{N}\vec{\omega}_{is}\mathbb{E}\{e_{s,k|k}e_{s,k|k}^T\}\Gamma^T$$
$$=\vec{\omega}_i\sum_{j=1}^{N}\vec{\omega}_{ij}\Gamma X_{j,k|k}\Gamma^T, \tag{23}$$

and

$$\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\ddot{\omega}_{ij}\ddot{\omega}_{is}\mathbb{E}\{x_{j,k}x_{s,k}^T\}\Gamma^T$$
$$\leq \frac{1}{2}\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\ddot{\omega}_{ij}\ddot{\omega}_{is}\mathbb{E}\{x_{j,k}x_{j,k}^T+x_{s,k}x_{s,k}^T\}\Gamma^T$$
$$=\frac{1}{2}\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\ddot{\omega}_{ij}\ddot{\omega}_{is}\mathbb{E}\{x_{j,k}x_{j,k}^T\}\Gamma^T$$
$$+\frac{1}{2}\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\ddot{\omega}_{ij}\ddot{\omega}_{is}\mathbb{E}\{x_{s,k}x_{s,k}^T\}\Gamma^T$$
$$=\frac{1}{2}\Gamma\sum_{s=1}^{N}\ddot{\omega}_{is}\sum_{j=1}^{N}\ddot{\omega}_{ij}\mathbb{E}\{x_{j,k}x_{j,k}^T\}\Gamma^T$$
$$+\frac{1}{2}\Gamma\sum_{j=1}^{N}\ddot{\omega}_{ij}\sum_{s=1}^{N}\ddot{\omega}_{is}\mathbb{E}\{x_{s,k}x_{s,k}^T\}\Gamma^T$$
$$=\ddot{\omega}_i\sum_{j=1}^{N}\ddot{\omega}_{ij}\Gamma\mathbb{E}\{x_{j,k}x_{j,k}^T\}\Gamma^T, \tag{24}$$

where $\vec{\omega}_i$ and $\ddot{\omega}_i$ are defined in (18). Moreover, it is s-traightforward to know that

$$\mathbb{E}\{x_{j,k}x_{j,k}^T\}\leq(1+\varrho_2)X_{j,k|k}+(1+\varrho_2^{-1})\hat{x}_{j,k|k}\hat{x}_{j,k|k}^T, \tag{25}$$

where $\varrho_2 > 0$ is a scalar. Substituting (25) into (24) results in

$$\Gamma\sum_{j=1}^{N}\sum_{s=1}^{N}\ddot{\omega}_{ij}\ddot{\omega}_{is}\mathbb{E}\{x_{j,k}x_{s,k}^T\}\Gamma^T$$
$$\leq\ddot{\omega}_i(1+\varrho_2)\sum_{j=1}^{N}\ddot{\omega}_{ij}\Gamma X_{j,k|k}\Gamma^T$$
$$+\ddot{\omega}_i(1+\varrho_2^{-1})\sum_{j=1}^{N}\ddot{\omega}_{ij}\Gamma\hat{x}_{j,k|k}\hat{x}_{j,k|k}^T\Gamma^T. \tag{26}$$

Similarly, we have the following inequalities:

$$\bar{\Gamma}\sum_{j=1}^{N}\sum_{s=1}^{N}\bar{\alpha}_{i,k}\omega_{ij}^{(1)}\omega_{is}^{(1)}\mathbb{E}\{x_{j,k}x_{s,k}^T\}\bar{\Gamma}^T$$
$$\leq\bar{\alpha}_{i,k}\omega_i^{(1)}(1+\varrho_3)\sum_{j=1}^{N}\omega_{ij}^{(1)}\bar{\Gamma}X_{j,k|k}\bar{\Gamma}^T$$
$$+\bar{\alpha}_{i,k}\omega_i^{(1)}(1+\varrho_3^{-1})\sum_{j=1}^{N}\omega_{ij}^{(1)}\bar{\Gamma}\hat{x}_{j,k|k}\hat{x}_{j,k|k}^T\bar{\Gamma}^T, \tag{27}$$
$$\bar{\Gamma}\sum_{j=1}^{N}\sum_{s=1}^{N}\omega_{ij}^{(2)}\omega_{is}^{(2)}\mathbb{E}\{x_{j,k}x_{s,k}^T\}\bar{\Gamma}^T$$
$$\leq\omega_i^{(2)}(1+\varrho_4)\sum_{j=1}^{N}\omega_{ij}^{(2)}\bar{\Gamma}X_{j,k|k}\bar{\Gamma}^T$$
$$+\omega_i^{(2)}(1+\varrho_4^{-1})\sum_{j=1}^{N}\omega_{ij}^{(2)}\bar{\Gamma}\hat{x}_{j,k|k}\hat{x}_{j,k|k}^T\bar{\Gamma}^T, \tag{28}$$

where ϱ_3 and ϱ_4 are positive constants, $\omega_i^{(1)}$ and $\omega_i^{(2)}$ are given in (18). Taking (23) and (26)-(28) into considera-tion, we obtain

$$X_{i,k+1|k}$$
$$\leq\sum_{j=1}^{N}\omega_{ij}^{'}\Gamma X_{j,k|k}\Gamma^T+\ddot{\omega}_i\bar{\alpha}_{i,k}(1-\bar{\alpha}_{i,k})(1+\varrho_2^{-1})$$
$$\times\sum_{j=1}^{N}\ddot{\omega}_{ij}\Gamma\hat{x}_{j,k|k}\hat{x}_{j,k|k}^T\Gamma^T+\sum_{j=1}^{N}\omega_{ij}^{''}\bar{\Gamma}X_{j,k|k}\bar{\Gamma}^T$$

$$+\sum_{j=1}^{N}\omega_{ij}^{'''}\bar{\Gamma}\hat{x}_{j,k|k}\hat{x}_{j,k|k}^{T}\bar{\Gamma}^{T}+B_{i,k}W_{i,k}B_{i,k}^{T}$$
$$+(1+\varrho_1^{-1})(A_{i,k}+F_{i,k}\Im_{i,k}L_{i,k})X_{i,k|k}$$
$$\times(A_{i,k}+F_{i,k}\Im_{i,k}L_{i,k})^{T}, \quad (29)$$

where $\omega_{ij}^{'}$, $\omega_{ij}^{''}$ and $\omega_{ij}^{'''}$ are denoted in (18).

Step 2: The recursion equation of upper bound matrix for estimation error covariance matrix $X_{i,k+1|k+1}$ is presented. Recalling the inequality in (20), one has

$$-\mathfrak{M}_1-\mathfrak{M}_1^{T}$$
$$\leq\varrho_5(I-\mathcal{K}_{i,k+1}\bar{\Pi}_{i,k+1}C_{i,k+1})X_{i,k+1|k}(I-\mathcal{K}_{i,k+1}$$
$$\times\bar{\Pi}_{i,k+1}C_{i,k+1})^{T}+\varrho_5^{-1}\mathcal{K}_{i,k+1}\mathbb{E}\{(\bar{y}_{i,k+1}-y_{i,k+1})$$
$$\times(\bar{y}_{i,k+1}-y_{i,k+1})^{T}\}\mathcal{K}_{i,k+1}^{T}, \quad (30)$$
$$\mathfrak{M}_2+\mathfrak{M}_2^{T}$$
$$\leq\varrho_6\mathcal{K}_{i,k+1}\mathbb{E}\{(\bar{y}_{i,k+1}-y_{i,k+1})(\bar{y}_{i,k+1}-y_{i,k+1})^{T}\}\mathcal{K}_{i,k+1}^{T}$$
$$+\varrho_6^{-1}\mathcal{K}_{i,k+1}\mathbb{E}\{\tilde{\Pi}_{i,k+1}C_{i,k+1}x_{i,k+1}x_{i,k+1}^{T}$$
$$\times C_{i,k+1}^{T}\tilde{\Pi}_{i,k+1}\}\mathcal{K}_{i,k+1}^{T}, \quad (31)$$
$$\mathfrak{M}_3+\mathfrak{M}_3^{T}$$
$$\leq\varrho_7\mathcal{K}_{i,k+1}\mathbb{E}\{(\bar{y}_{i,k+1}-y_{i,k+1})(\bar{y}_{i,k+1}-y_{i,k+1})^{T}\}\mathcal{K}_{i,k+1}^{T}$$
$$+\varrho_7^{-1}\mathcal{K}_{i,k+1}V_{i,k+1}\mathcal{K}_{i,k+1}^{T}, \quad (32)$$

where ϱ_5, ϱ_6 and ϱ_7 are positive scalars. Substituting (30)-(32) into (14) leads to

$$X_{i,k+1|k+1}$$
$$\leq(1+\varrho_5)(I-\mathcal{K}_{i,k+1}\bar{\Pi}_{i,k+1}C_{i,k+1})X_{i,k+1|k}$$
$$\times(I-\mathcal{K}_{i,k+1}\bar{\Pi}_{i,k+1}C_{i,k+1})^{T}+(1+\varrho_5^{-1}+\varrho_6+\varrho_7)$$
$$\times\mathcal{K}_{i,k+1}\mathbb{E}\{(\bar{y}_{i,k+1}-y_{i,k+1})(\bar{y}_{i,k+1}-y_{i,k+1})^{T}\}\mathcal{K}_{i,k+1}^{T}$$
$$+(1+\varrho_6^{-1})\mathcal{K}_{i,k+1}\mathbb{E}\{\tilde{\Pi}_{i,k+1}C_{i,k+1}x_{i,k+1}x_{i,k+1}^{T}C_{i,k+1}^{T}$$
$$\times\tilde{\Pi}_{i,k+1}\}\mathcal{K}_{i,k+1}^{T}+(1+\varrho_7^{-1})\mathcal{K}_{i,k+1}V_{i,k+1}\mathcal{K}_{i,k+1}^{T}. \quad (33)$$

In light of (5), we arrive at

$$\mathbb{E}\{(\bar{y}_{i,k+1}-y_{i,k+1})(\bar{y}_{i,k+1}-y_{i,k+1})^{T}\}\leq\delta_i I. \quad (34)$$

According to Lemma 2, the following relationship is derived:

$$\mathbb{E}\{\tilde{\Pi}_{i,k+1}C_{i,k+1}x_{i,k+1}x_{i,k+1}^{T}C_{i,k+1}^{T}\tilde{\Pi}_{i,k+1}\}$$
$$=\Xi_{i,k+1}\circ\mathbb{E}\{C_{i,k+1}x_{i,k+1}x_{i,k+1}^{T}C_{i,k+1}^{T}\}, \quad (35)$$

where $\Xi_{i,k+1}$ is denoted in (18). Similar to (25), one has

$$\mathbb{E}\{x_{i,k+1}x_{i,k+1}^{T}\}$$
$$=\mathbb{E}\{(e_{i,k+1|k}+\hat{x}_{i,k+1|k})(e_{i,k+1|k}+\hat{x}_{i,k+1|k})^{T}\}$$
$$\leq(1+\varrho_8)X_{i,k+1|k}+(1+\varrho_8^{-1})\hat{x}_{i,k+1|k}\hat{x}_{i,k+1|k}^{T}$$
$$:=\Psi_{i,k+1}, \quad (36)$$

where ϱ_8 is a positive scalar. Therefore, we have

$$\mathbb{E}\{\tilde{\Pi}_{i,k+1}C_{i,k+1}x_{i,k+1}x_{i,k+1}^{T}C_{i,k+1}^{T}\tilde{\Pi}_{i,k+1}\}$$
$$\leq\Xi_{i,k+1}\circ(C_{i,k+1}\Psi_{i,k+1}C_{i,k+1}^{T}). \quad (37)$$

Taking (34) and (37) into consideration results in

$$X_{i,k+1|k+1}$$
$$\leq(1+\varrho_5)(I-\mathcal{K}_{i,k+1}\bar{\Pi}_{i,k+1}C_{i,k+1})X_{i,k+1|k}$$
$$\times(I-\mathcal{K}_{i,k+1}\bar{\Pi}_{i,k+1}C_{i,k+1})^{T}$$
$$+\mathcal{K}_{i,k+1}\Big\{(1+\varrho_5^{-1}+\varrho_6+\varrho_7)\delta_i I+(1+\varrho_7^{-1})V_{i,k+1}$$
$$+(1+\varrho_6^{-1})\Xi_{i,k+1}\circ(C_{i,k+1}\Psi_{i,k+1}C_{i,k+1}^{T})\Big\}\mathcal{K}_{i,k+1}^{T}. \quad (38)$$

Combining (15)-(16) and (29) with (38) implies that $X_{i,k+1|k+1}\leq\mathcal{X}_{i,k+1|k+1}$.

Step 3: Finally, we are in a position to provide the parameterization of the desired estimator gain matrix that minimizes $\mathcal{X}_{i,k+1|k+1}$. Taking the partial derivative of $\mathrm{tr}(\mathcal{X}_{i,k+1|k+1})$ w.r.t the estimator parameter $\mathcal{K}_{i,k+1}$, one obtains

$$\frac{\partial\mathrm{tr}(\mathcal{X}_{i,k+1|k+1})}{\partial\mathcal{K}_{i,k+1}}$$
$$=-2(1+\varrho_5)(I-\mathcal{K}_{i,k+1}\bar{\Pi}_{i,k+1}C_{i,k+1})\mathcal{X}_{i,k+1|k}C_{i,k+1}^{T}$$
$$\times\bar{\Pi}_{i,k+1}+2\mathcal{K}_{i,k+1}\Big\{(1+\varrho_5^{-1}+\varrho_6+\varrho_7)$$
$$\times\delta_i I+(1+\varrho_7^{-1})V_{i,k+1}+(1+\varrho_6^{-1})\Xi_{i,k+1}$$
$$\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^{T})\Big\}. \quad (39)$$

Letting $\frac{\partial\mathrm{tr}(\mathcal{X}_{i,k+1|k+1})}{\partial\mathcal{K}_{i,k+1}}=0$, we obtain the following estimator gain

$$\mathcal{K}_{i,k+1}$$
$$=(1+\varrho_5)\mathcal{X}_{i,k+1|k}C_{i,k+1}^{T}\bar{\Pi}_{i,k+1}\Big\{(1+\varrho_5)\bar{\Pi}_{i,k+1}C_{i,k+1}$$
$$\times\mathcal{X}_{i,k+1|k}C_{i,k+1}^{T}\bar{\Pi}_{i,k+1}+(1+\varrho_5^{-1}+\varrho_6+\varrho_7)\delta_i I$$
$$+(1+\varrho_7^{-1})V_{i,k+1}+(1+\varrho_6^{-1})\Xi_{i,k+1}$$
$$\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^{T})\Big\}^{-1},$$

which is the same as the one in (19). Consequently, the assertions in this theorem are true.

Remark 4 *The randomly switching topologies are modelled in (1) to reflect the stochastic fluctuations and the inevitable modelling errors with regard to network topology structure. In particular, the Bernoulli distributed variables $\alpha_{i,k}$ are used to model this phenomenon, where the parameters $\bar{\alpha}_{i,k}$ in (3) representing the occurrence probabilities of the randomly switching topologies could be obtained according to the extensive statistical tests. It should be noted that the existence of the randomly switching topologies brings essential difficulties when analyzing the dynamics behaviours of the complex networks and estimating the state of each node. For example, there is a need to better understand and appropriately characterize* *the randomly switching topologies. Moreover, the effects caused by randomly switching topologies should be revealed when designing the state estimation method and those effects should be compensated efficiently in the proposed estimation algorithm. In order to overcome those difficulties and improve the estimation accuracy, additional effort has been made and some terms involved the corresponding information regarding randomly switching topologies (e.g.* $\bar{\alpha}_{i,k}$, $\omega_{ij}^{'}$, $\omega_{ij}^{''}$, $\omega_{ij}^{'''}$, $\vec{\omega}_i$, $\ddot{\omega}_i$, $\omega_i^{(1)}$, $\omega_i^{(2)}$ *and* Γ*) have been explicitly reflected in the developed state estimation scheme as shown in Theorem 3.*

Remark 5 *The solutions to recursive matrix difference equations in (15) and (16) represent the covariance matrices of the prediction error and estimation error, respectively. Due to the existence of the different type noises, randomly switching topologies, degraded measurements and event-triggered communication protocol, it is not difficult to find the corresponding positive definite solutions, which is same with the case as made in the classical Kalman filtering method. On the other hand, it is worth mentioning that the positive scalars* $\gamma_{i,k}$ *are introduced when dealing with the high-order terms of the Taylor series expansions. During the algorithm implementation, we can fix the values of* $\gamma_{i,k}$ *and adjust the values thereafter to ensure the feasibility of the constraint (17), thereby enhancing the flexibility of the proposed event-based estimation algorithm.*

To end this section, the newly developed event-triggered recursive state estimation (ETRST) algorithm is summarized.

Algorithm ETRST :

Step I: Initialize the corresponding conditions and set other parameters.

Step II: Calculate the prediction estimation $\hat{x}_{i,k+1|k}$ according to (6).

Step III: Compute the prediction error covariance $\mathcal{X}_{i,k+1|k}$ by using (15) and obtain the estimator parameter matrix $\mathcal{K}_{i,k+1}$ via (19).

Step IV: The estimation error covariance $\mathcal{X}_{i,k+1|k+1}$ is derived by (16).

Step V: Calculate the state estimation $\hat{x}_{i,k+1|k+1}$ based on (7).

Step VI: Setting $k = k+1$, and go to Step II.

Remark 6 *To facilitate further implementation, the new ETRST algorithm is summarized above regarding the addressed state estimation problem. It can be observed that the proposed algorithm has the recursive feature since the upper bound matrices of the prediction error covariance and the estimation error covariance (i.e.,* $\mathcal{X}_{i,k+1|k}$ *and* $\mathcal{X}_{i,k+1|k+1}$ *in (15)-(16)) are forward matrix difference equations as the time goes. Hence, the new ETRST algorithm is applicable for online implementations in real-time updating environment, which performs a prevailing advantage of main results.*

4 Performance Analysis

This section is devoted to the monotonicity analysis of our main results, that is, we are interested in revealing the relationship between $\mathrm{tr}\{\mathcal{X}_{i,k+1|k+1}\}$ and the probabilities $\bar{\pi}_{i,k+1}$.

To proceed, suppose that $\bar{\pi}_{i,k+1}^{(l)} = \bar{\pi}_{i,k+1}$ $(l = 1,2,\cdots,m)$. Thus, it is easy to know that $\bar{\Pi}_{i,k+1} = \bar{\pi}_{i,k+1}I$ and $\Xi_{i,k+1} = \vec{\pi}_{i,k+1}I$ with $\vec{\pi}_{i,k+1} = \bar{\pi}_{i,k+1}(1-\bar{\pi}_{i,k+1})$. In what follows, the relationship between $\mathrm{tr}\{\mathcal{X}_{i,k+1|k+1}\}$ and $\bar{\pi}_{i,k+1}$ is established via rather direct algebraic calculations.

Theorem 4 *It can be shown that* $\mathrm{tr}\{\mathcal{X}_{i,k+1|k+1}\}$ *is non-increasing when the occurrence probability* $\bar{\pi}_{i,k+1}$ *increases.*

Proof: To facilitate further developments, set

$$\begin{aligned}\Upsilon_{i,k+1} &= (1+\varrho_5^{-1}+\varrho_6+\varrho_7)\delta_i I + (1+\varrho_7^{-1})V_{i,k+1}\\ &\quad +(1+\varrho_6^{-1})\Xi_{i,k+1}\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T),\\ \Theta_{i,k+1} &= (1+\varrho_5)\bar{\Pi}_{i,k+1}C_{i,k+1}\mathcal{X}_{i,k+1|k}C_{i,k+1}^T\bar{\Pi}_{i,k+1}\\ &\quad +\Upsilon_{i,k+1}.\end{aligned}$$

Then, according to the above notations, (16) can be rewritten as follows:

$$\begin{aligned}&\mathcal{X}_{i,k+1|k+1}\\ &= (1+\varrho_5)\mathcal{X}_{i,k+1|k} - (1+\varrho_5)^2\mathcal{X}_{i,k+1|k}C_{i,k+1}^T\bar{\Pi}_{i,k+1}\\ &\quad \times\Theta_{i,k+1}^{-1}\bar{\Pi}_{i,k+1}C_{i,k+1}\mathcal{X}_{i,k+1|k} + \Big[\mathcal{K}_{i,k+1} - (1+\varrho_5)\\ &\quad \times\mathcal{X}_{i,k+1|k}C_{i,k+1}^T\bar{\Pi}_{i,k+1}\Theta_{i,k+1}^{-1}\Big]\Theta_{i,k+1}\Big[\mathcal{K}_{i,k+1}\\ &\quad -(1+\varrho_5)\mathcal{X}_{i,k+1|k}C_{i,k+1}^T\bar{\Pi}_{i,k+1}\Theta_{i,k+1}^{-1}\Big]^T.\end{aligned}$$

Notice that $\mathcal{K}_{i,k+1} = (1+\varrho_5)\mathcal{X}_{i,k+1|k}C_{i,k+1}^T\bar{\Pi}_{i,k+1}\Theta_{i,k+1}^{-1}$, we have

$$\begin{aligned}\mathcal{X}_{i,k+1|k+1} &= (1+\varrho_5)\mathcal{X}_{i,k+1|k} - (1+\varrho_5)^2\mathcal{X}_{i,k+1|k}\\ &\quad \times C_{i,k+1}^T\bar{\Pi}_{i,k+1}\Theta_{i,k+1}^{-1}\bar{\Pi}_{i,k+1}C_{i,k+1}\mathcal{X}_{i,k+1|k}.\end{aligned}$$

Secondly, taking the partial derivative of $\mathrm{tr}\{\mathcal{X}_{i,k+1|k+1}\}$ w.r.t $\bar{\pi}_{i,k+1}$ results in

$$\begin{aligned}&\frac{\mathrm{d}\mathrm{tr}\{\mathcal{X}_{i,k+1|k+1}\}}{\mathrm{d}\bar{\pi}_{i,k+1}}\\ &= \frac{\mathrm{d}\mathrm{tr}}{\mathrm{d}\bar{\pi}_{i,k+1}}\Big\{(1+\varrho_5)\mathcal{X}_{i,k+1|k} - (1+\varrho_5)^2\bar{\pi}_{i,k+1}^2\mathcal{X}_{i,k+1|k}\\ &\quad \times C_{i,k+1}^T\big[(1+\varrho_5)\bar{\pi}_{i,k+1}^2C_{i,k+1}\mathcal{X}_{i,k+1|k}C_{i,k+1}^T\\ &\quad +(1+\varrho_6^{-1})\vec{\pi}_{i,k+1}I\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T)\\ &\quad +(1+\varrho_5^{-1}+\varrho_6+\varrho_7)\delta_i I + (1+\varrho_7^{-1})V_{i,k+1}\big]^{-1}\\ &\quad \times C_{i,k+1}\mathcal{X}_{i,k+1|k}\Big\}\\ &= \mathrm{tr}\Big\{-2(1+\varrho_5)^2\bar{\pi}_{i,k+1}\mathcal{X}_{i,k+1|k}C_{i,k+1}^T\big[(1+\varrho_5)\bar{\pi}_{i,k+1}^2\\ &\quad \times C_{i,k+1}\mathcal{X}_{i,k+1|k}C_{i,k+1}^T + (1+\varrho_6^{-1})\vec{\pi}_{i,k+1}I\\ &\quad \circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T) + (1+\varrho_5^{-1}+\varrho_6+\varrho_7)\delta_i I\\ &\quad +(1+\varrho_7^{-1})V_{i,k+1}\big]^{-1}C_{i,k+1}\mathcal{X}_{i,k+1|k} + (1+\varrho_5)^2\bar{\pi}_{i,k+1}^2\end{aligned}$$

$$
\times\mathcal{X}_{i,k+1|k}C_{i,k+1}^T\big[(1+\varrho_5)\bar{\pi}_{i,k+1}^2C_{i,k+1}\mathcal{X}_{i,k+1|k}C_{i,k+1}^T
+(1+\varrho_6^{-1})\vec{\pi}_{i,k+1}I\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T)
+(1+\varrho_5^{-1}+\varrho_6+\varrho_7)\delta_iI+(1+\varrho_7^{-1})V_{i,k+1}\big]^{-1}
\times\big[2(1+\varrho_5)\bar{\pi}_{i,k+1}C_{i,k+1}\mathcal{X}_{i,k+1|k}C_{i,k+1}^T+(1+\varrho_6^{-1})
\times(1-2\bar{\pi}_{i,k+1})I\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T)\big]
\times\big[(1+\varrho_5)\bar{\pi}_{i,k+1}^2C_{i,k+1}\mathcal{X}_{i,k+1|k}C_{i,k+1}^T+(1+\varrho_6^{-1})
\times\vec{\pi}_{i,k+1}I\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T)+(1+\varrho_5^{-1}+\varrho_6
+\varrho_7)\delta_iI+(1+\varrho_7^{-1})V_{i,k+1}\big]^{-1}C_{i,k+1}\mathcal{X}_{i,k+1|k}\Big\}. \quad (40)
$$

Next, one has

$$
\frac{\mathrm{dtr}\{\mathcal{X}_{i,k+1|k+1}\}}{\mathrm{d}\bar{\pi}_{i,k+1}}
\le\mathrm{tr}\Big\{-2(1+\varrho_5)^2\bar{\pi}_{i,k+1}\mathcal{X}_{i,k+1|k}C_{i,k+1}^T\big[(1+\varrho_5)\bar{\pi}_{i,k+1}^2
\times C_{i,k+1}\mathcal{X}_{i,k+1|k}C_{i,k+1}^T+(1+\varrho_6^{-1})\vec{\pi}_{i,k+1}I
\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T)+(1+\varrho_5^{-1}+\varrho_6+\varrho_7)\delta_iI
+(1+\varrho_7^{-1})V_{i,k+1}\big]^{-1}C_{i,k+1}\mathcal{X}_{i,k+1|k}
+(1+\varrho_5)^2\bar{\pi}_{i,k+1}\mathcal{X}_{i,k+1|k}C_{i,k+1}^T
\times\big[(1+\varrho_5)\bar{\pi}_{i,k+1}^2C_{i,k+1}\mathcal{X}_{i,k+1|k}C_{i,k+1}^T+(1+\varrho_6^{-1})
\times\vec{\pi}_{i,k+1}I\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T)
+(1+\varrho_5^{-1}+\varrho_6+\varrho_7)\delta_iI+(1+\varrho_7^{-1})V_{i,k+1}\big]^{-1}
\times\big[2(1+\varrho_5)\bar{\pi}_{i,k+1}^2C_{i,k+1}\mathcal{X}_{i,k+1|k}C_{i,k+1}^T+2(1+\varrho_6^{-1})
\times\vec{\pi}_{i,k+1}I\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T)
+2(1+\varrho_5^{-1}+\varrho_6+\varrho_7)\delta_iI+2(1+\varrho_7^{-1})V_{i,k+1}
-(1+\varrho_6^{-1})\bar{\pi}_{i,k+1}I\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T)\big]
\times\big[(1+\varrho_5)\bar{\pi}_{i,k+1}^2C_{i,k+1}\mathcal{X}_{i,k+1|k}C_{i,k+1}^T+(1+\varrho_6^{-1})
\times\vec{\pi}_{i,k+1}I\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T)+(1+\varrho_5^{-1}+\varrho_6
+\varrho_7)\delta_iI+(1+\varrho_7^{-1})V_{i,k+1}\big]^{-1}C_{i,k+1}\mathcal{X}_{i,k+1|k}\Big\}. \quad (41)
$$

By removing the zero term in (41), we arrive at

$$
\frac{\mathrm{dtr}\{\mathcal{X}_{i,k+1|k+1}\}}{\mathrm{d}\bar{\pi}_{i,k+1}}
\le-(1+\varrho_5)^2(1+\varrho_6^{-1})\mathrm{tr}\Big\{\mathcal{X}_{i,k+1|k}C_{i,k+1}^T
\times\big[(1+\varrho_5)\bar{\pi}_{i,k+1}^2C_{i,k+1}\mathcal{X}_{i,k+1|k}C_{i,k+1}^T+(1+\varrho_6^{-1})
\times\vec{\pi}_{i,k+1}I\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T)+(1+\varrho_5^{-1}+\varrho_6
+\varrho_7)\delta_iI+(1+\varrho_7^{-1})V_{i,k+1}\big]^{-1}\bar{\pi}_{i,k+1}^2I
\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T)\big[(1+\varrho_5)\bar{\pi}_{i,k+1}^2C_{i,k+1}
\times\mathcal{X}_{i,k+1|k}C_{i,k+1}^T+(1+\varrho_6^{-1})\vec{\pi}_{i,k+1}I
\circ(C_{i,k+1}\bar{\Psi}_{i,k+1}C_{i,k+1}^T)+(1+\varrho_5^{-1}+\varrho_6+\varrho_7)\delta_iI
+(1+\varrho_7^{-1})V_{i,k+1}\big]^{-1}C_{i,k+1}\mathcal{X}_{i,k+1|k}\Big\}. \quad (42)
$$

Finally, it is easy to check that

$$
\frac{\mathrm{dtr}\{\mathcal{X}_{i,k+1|k+1}\}}{\mathrm{d}\bar{\pi}_{i,k+1}}\le 0,
$$

because the term $\mathrm{tr}\{\cdot\}$ in (42) is non-negative. Consequently, it can be concluded that $\mathrm{tr}\{\mathcal{X}_{i,k+1|k+1}\}$ is non-increasing provided that the occurrence probability $\bar{\pi}_{i,k+1}$ increases.

Remark 7 *In Theorem 4, special effort has been made to further reveal the changes of estimation accuracy accompanying with the degraded measurements, that is, the monotonicity analysis between* $\mathrm{tr}\{\mathcal{X}_{i,k+1|k+1}\}$ *and* $\bar{\pi}_{i,k+1}$ *is provided from theoretical perspective. According to the theoretical analysis in Theorem 4, it can be observed that the term* $\mathrm{tr}\{\mathcal{X}_{i,k+1|k+1}\}$ *is non-increasing if the value of occurrence probability* $\bar{\pi}_{i,k+1}$ *increases. Apparently, when the occurrence probability* $\bar{\pi}_{i,k+1}$ *increases in reality, it means that the measurement data is more likely to be transmitted safely to the remote estimator, thereby improving the estimation accuracy, and this is consistent with the practical engineering viewpoint.*

Remark 8 *So far, we have proposed a new event-based state estimation approach with theoretical analysis for addressed nonlinear time-varying dynamical networks subject to randomly switching topologies and multiple missing measurements. It is worth noting that the available information of multiple missing measurements, randomly switching topologies and event-triggered communication protocol has been clearly reflected in main results. To be more specific,* $\bar{\alpha}_{i,k}$ *refers to the randomly switching topologies,* $\bar{\pi}_{i,k}^{(l)}$ *is there for the multiple missing measurements, and* δ_i *corresponds to the event-triggered communication scheme.*

5 A Simulation Example

In this section, some simulations with comparisons are presented to show the feasibility and usefulness of the proposed event-triggered recursive estimation strategy.

Consider the nonlinear coupled dynamical networks (1)-(2) with system parameters given by:

$$
B_{1,k}=\begin{bmatrix}0.4-0.1\sin(0.2k) & 0.4\end{bmatrix}^T,
$$
$$
B_{2,k}=\begin{bmatrix}-0.5 & -0.79\end{bmatrix}^T,\quad B_{3,k}=\begin{bmatrix}0.2 & 0.95\end{bmatrix}^T,
$$
$$
C_{1,k}=\begin{bmatrix}0.2 & -1.7\end{bmatrix},\quad C_{3,k}=\begin{bmatrix}1 & 1.2\end{bmatrix},
$$
$$
C_{2,k}=\begin{bmatrix}-1.2-0.1\cos(0.6k) & 1.5\end{bmatrix},
$$
$$
W^{(1)}=\begin{bmatrix}-0.26 & 0.12 & 0.12\\ 0.12 & -0.26 & 0.12\\ 0.12 & 0.12 & -0.26\end{bmatrix},\quad \Gamma=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix},
$$
$$
W^{(2)}=\begin{bmatrix}-0.36 & 0.15 & 0.15\\ 0.15 & -0.36 & 0.15\\ 0.15 & 0.15 & -0.36\end{bmatrix},\quad \bar{\Gamma}=\begin{bmatrix}0.1 & 0\\ 0 & 0.1\end{bmatrix}.
$$

The nonlinearity $f(x_{i,k})$ is described by

$$f(x_{i,k}) = \begin{bmatrix} -0.12x_{i,k}^1 + 0.25x_{i,k}^2 + 0.068\sin(x_{i,k}^1 x_{i,k}^2) \\ 1.15x_{i,k}^1 + 0.825x_{i,k}^2 - 0.0485\cos(x_{i,k}^1 x_{i,k}^2) \end{bmatrix},$$

where $x_{i,k} = \begin{bmatrix} x_{i,k}^1 & x_{i,k}^2 \end{bmatrix}^T$ $(i = 1, 2, 3)$ depicts the system state of the i-th node.

To implement the developed recursive state estimation method, the mathematical expectations of the initial values are chosen as $\bar{x}_{1,0} = \begin{bmatrix} -0.3 & -0.2 \end{bmatrix}^T$, $\bar{x}_{2,0} = \bar{x}_{3,0} = \begin{bmatrix} 0.1 & 0.4 \end{bmatrix}^T$, $\hat{x}_{1,0|0} = \begin{bmatrix} -1.3 & -1.2 \end{bmatrix}^T$, $\hat{x}_{2,0|0} = \hat{x}_{3,0|0} = \begin{bmatrix} -0.9 & -0.6 \end{bmatrix}^T$ and $\mathcal{X}_{i,0|0} = 2.5I_2$ $(i = 1, 2, 3)$. Other parameters are selected as $\varrho_1 = 0.1$, $\varrho_2 = \varrho_3 = \varrho_4 = 1$, $\varrho_5 = \varrho_6 = \varrho_7 = 0.1$, $\varrho_8 = 0.01$, $W_{1,k} = 0.2$, $W_{2,k} = 0.1$, $W_{3,k} = V_{1,k} = 0.02$, $V_{2,k} = V_{3,k} = 0.01$, $L_{i,k} = 0.1I_2$, $F_{i,k} = 0.1I_2$, $\bar{\alpha}_{i,k} = 0.75$ and $\bar{\pi}_{i,k}^{(1)} = 0.95$. Then, based on the result in Theorem 3, the state estimation method can be implemented and the estimator gain can be obtained recursively.

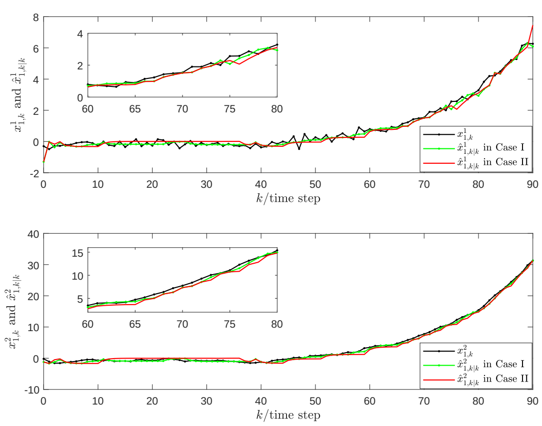

Fig. 1. The response trajectories of $x_{1,k}^s$ and $\hat{x}_{1,k|k}^s$ $(s = 1, 2)$.

To reflect the impact from the event-triggered strategy onto the estimation performance, we provide the comparative simulations concerning on the estimation performance under different triggered thresholds, i.e., Case I: $\delta_1 = \delta_2 = \delta_3 = 1$ and Case II: $\delta_1 = \delta_2 = \delta_3 = 4$. The related simulations are provided in Figs. 1-7, where Figs. 1-3 plot the trajectories of state $x_{i,k}$ and the estimation $\hat{x}_{i,k|k}$. In particular, some sub-figures are added in Figs. 1-3 and the actual scale of each sub-figure is chosen from 60 to 80 for the abscissa axis. From the related simulations, we can see that the estimation accuracy is better when the triggered threshold is small. The reason is that more measurements can be obtained in the estimator side, thereby improving the estimation performance in this case. Fig. 4 shows the log(MSE) in Cases I-II, where MSE stands for the mean square error of the node state estimation. Note that the chances of released information during the network transmission could be increased when the triggered thresholds decrease as in Case I, thus the estimation accuracy of the proposed algorithm is improved. It is easy to see that this fact has been clearly shown as provided in Figs. 1-4.

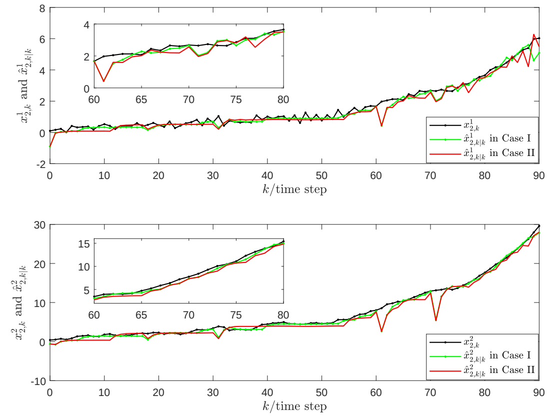

Fig. 2. The response trajectories of $x_{2,k}^s$ and $\hat{x}_{2,k|k}^s$ $(s = 1, 2)$.

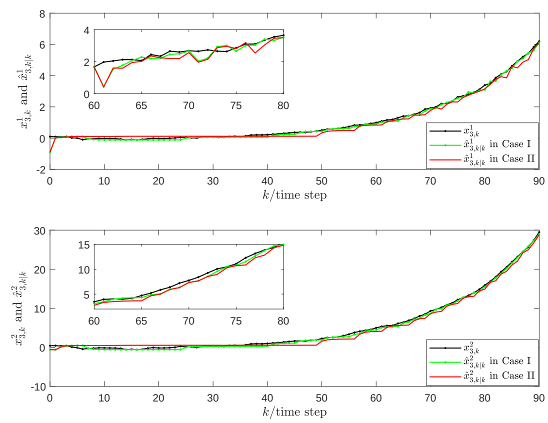

Fig. 3. The response trajectories of $x_{3,k}^s$ and $\hat{x}_{3,k|k}^s$ $(s = 1, 2)$.

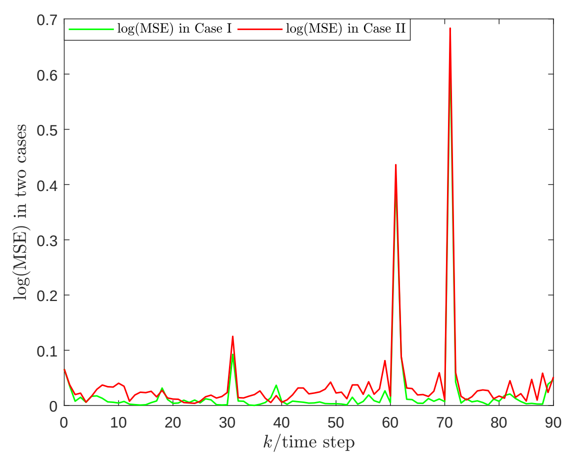

Fig. 4. log(MSE) in two cases.

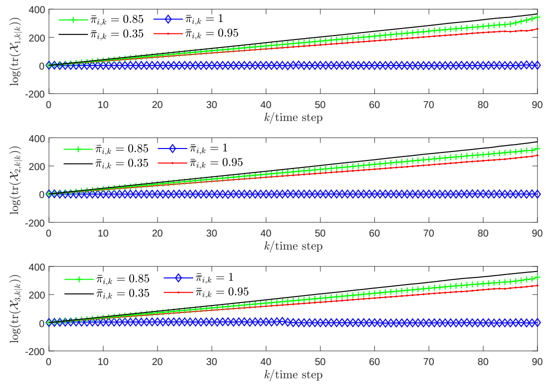

Fig. 5. $\log(\mathrm{tr}(\mathcal{X}_{i,k|k}))$ under different $\bar{\pi}_{i,k}$ $(i = 1, 2, 3)$.

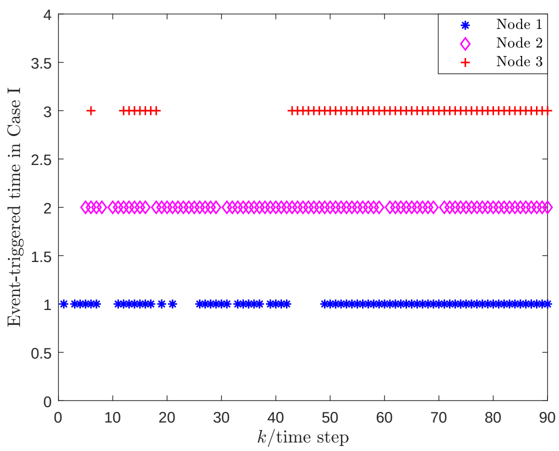

Fig. 6. The event-triggered time in Case I.

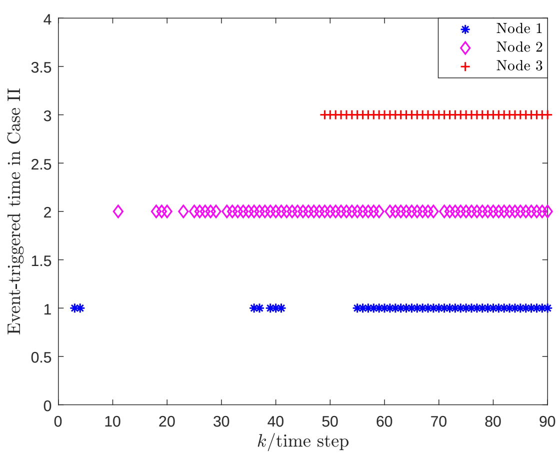

Fig. 7. The event-triggered time in Case II.

On the other hand, in order to further discuss the effects from the degraded measurements, $\log(\mathrm{tr}(\mathcal{X}_{i,k|k}))$ is plotted regarding missing measurements under different occurrence probabilities, i.e., $\bar{\pi}_{i,k}$ are 0.35, 0.85, 0.95 and 1, respectively. The corresponding simulation result is presented in Fig. 5. It can be seen from Fig. 5 that $\log(\mathrm{tr}(\mathcal{X}_{i,k|k}))$ increases when $\bar{\pi}_{i,k}$ decreases, which represents the fact that the estimation accuracy of the proposed estimation algorithm becomes worse when more useful information is lost. According to this comparison under different occurrence probabilities, we can further verify the assertion mentioned in Theorem 4. Moreover, for all nodes, Figs. 6 and 7 describe the event-triggered time in Cases I-II. Now, it can be observed that the above simulations further illustrate the effectiveness of newly variance-constrained state estimation algorithm.

Remark 9 *It is worthwhile to point out that the nonlinear time-varying dynamical networks established in the paper are more comprehensive than the existing ones. Moreover, the addressed event-based optimized state estimation problem is new, which fully handles the induced effects from randomly switching topologies, multiple missing measurements and event-triggered communication mechanism in a unified framework. After the extensive literature search, we find that there has been no existing state estimation algorithms applicable for the same estimation problem addressed in the paper. Hence, we further emphasize the main advantages of the proposed state estimation method from the following three aspects: 1) the time-varying dynamical network model is quite comprehensive that reflects the above mentioned three phenomena, thereby better depicting the reality; 2) the newly designed state estimation scheme is in a distributed way without resorting the state argumentation method, which is not increasing the computational burdens; and 3) both the theoretical analysis and comparative experiments are provided to reveal the inherent relationship between the estimation accuracy and the different occurrence probabilities of degraded measurements. As shown in the simulation comparisons, it can be seen that the newly developed estimation scheme provides a satisfactory performance. Moreover, it is worthwhile to mention that the new estimation method can estimate the original system state well as the time goes, which is irrespective of the stability of the original system. Hence, the presented event-triggered estimation strategy enriches the method on handling the state estimation problem under variance constraint for time-varying dynamical networks over networked communications.*

6 Conclusions

This paper has investigated the recursive state estimation problem under variance-constraint for a class of nonlinear coupled dynamical networks subject to randomly switching topologies and degraded measurements. The phenomenon of multiple missing measurements has been characterized by means of a sequence of Bernoulli distributed variables. In addition, the randomly switching topologies have been modelled by using the random variable to reflect the randomly changeable topology characteristics of the coupled complex networks. Subsequently, a new time-varying state estimation method has been given such that, for all randomly switching topologies, event-triggered transmission protocol as well as missing measurements, the expression equation of minimized upper bound for the estimation error covariance matrix has been established and the desired state estimator gain has been proposed via the solutions to some recursive matrix difference equations. Besides, the theoretical proof has been given to discuss the monotonicity relationship with regard to the missing probability and the upper bound matrix. Finally, the validity of presented event-triggered estimation approach has been illustrated by some comparative simulations. Further research extensions include the discussions on the event-based time-varying estimation problems for semi-Markovian jump dynamical networks as in [17] and the estimation problem dealing with the round-robin communication protocol as in [6, 44].

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