IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received March 26, 2019, accepted April 24, 2019, date of publication May 3, 2019, date of current version May 16, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2914738

Disturbance Attenuation and Rejection for Nonlinear Uncertain Switched Systems Subject to Input Saturation

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This work was supported in part by the National Science Foundation of China under Grant 61703233, Grant 61773144, and Grant 61690212, in part by the Postdoctoral Science Foundation of China under Grant 2016M602112, and in part by the Natural Science Foundation of Shandong Province under Grant ZR2016FQ09.

ABSTRACT This paper studies the problem of disturbance attenuation and rejection for switched systems with nonlinear uncertainty and input saturation via composite anti-disturbance control technique, in which the exosystem-generated disturbance and H_2-norm-bounded disturbance are considered. For switched systems, the switching law and input saturation increase the difficulty for the design of the disturbance observer and the composite control scheme. A switching disturbance observer is designed to obtain the estimation of the matched disturbance, and a novel switching composite controller is further constructed based on the estimated value. By proposing a state-dependent switching law and utilizing the multiple Lyapunov function technology, the criteria are presented to ensure the local asymptotic stability with an H_∞ performance level for the closed-loop system. Furthermore, two optimal algorithms of the design of the controller are put forward to maximize the estimation of the domain of attraction of the closed-loop and the upper bound on the H_2-norm of the disturbance, respectively. The effectiveness of the proposed technique is illustrated via the numerical examples.

INDEX TERMS Switched systems, nonlinear uncertainties, multiple disturbances, input saturation, composite anti-disturbance control.

I. INTRODUCTION

As we all know that disturbance and uncertainty are important factors resulting in the performance degradation or stability loss for the controlled systems, which are inevitable in the actual engineering systems due in particular to load changing, external surroundings and modeling error [1]. Therefore, the Disturbance attenuation and rejection for uncertain systems become crucial in practice, which can be efficiently achieved by some classical control techniques, such as robust adaptive control [2], robust sliding mode control [3], robust H_∞ control [4], and so on. As one class of the advanced control methods, composite anti-disturbance control based on disturbance observation and compensation has attracted great attentions recently [5]–[8]. In [5], the problem of disturbance observer based control for a nonlinear uncertain system with an exosystem generated disturbance is investigated, and both reduced-order and full-order observers are designed for the composite system based on whether states of a system be available or not. Based on the above results, some new anti-disturbance control schemes are investigated for sorts and kinds of the controlled systems with multiple disturbances. In [6], a composite control strategy based on extended state observer is given for a class of nonlinear systems with mismatching disturbances, in which disturbance compensation gain matrix has been added in contrast to the composite controller in [5]. In [9], a composite hierarchical anti-disturbance control (CHADC) scheme has been put forward for the sys-

The associate editor coordinating the review of this manuscript and approving it for publication was Bora Onat.

tem, in which multiple disturbances are classified and modeled respectively. In recent ten years, the CHADC technique has been widely researched and generalized [10]–[14].

Due to physical and technical constraint of the input channel of the controlled systems, the phenomena of input saturation cannot be avoided in practical engineering which generally bring adverse effects of control performance [15]. Therefore, it is of great importance to consider the existence of saturation in the control process. For the last decades, there have existed several fundamental methods proposed for tackling control problem of the system subject to saturation, mainly including anti-windup compensation [16], "one-step" design method [17] and model predictive control [18]. Generally, saturation nonlinearity is treated locally as sector nonlinearity model or polytopic differential inclusion [15], [19].

Switched system is constituted by a family of continuous-time subsystems (or discrete-time subsystems) together with a switching signal that orchestrates the switching between them, which is widely used to model the physical systems with switching features, such as mechanical systems, aircraft and air traffic control systems, power and electronics systems [20]. For the study of control problem of the switched systems, not only the control scheme but also switching signal needs to be designed [21]–[26]. Recently, more attentions have been paid for the control problems of switched systems subject to input saturation [27]–[32]. Based on the minimum dwell time switching, a synthesis method of saturated feedbacks has been proposed, in which some sufficient design conditions for stabilizing controller are presented in term of linear matrix inequalities in [28]. In [29] and [21], the finite-time control problems are considered for the switched systems with saturating inputs, and event-triggered control for switched systems with saturating actuators is studied in [31]. In [32], the synthesis of feedback control laws to achieve disturbance attenuation and rejection has been studied for a singular switched systems subject to input saturation and H_2-norm bounded disturbance. Although many advanced control methods have been developed for the switched systems with saturation and single disturbance, the adverse effects from various disturbances have not been full considered. When the switched system is with uncertainties, input saturation and multiple disturbances, the system should be more complicated and the design of control schemes would be more difficult due to the interconnection between switching, saturation on whole input channel and multiple disturbances.

Motivated by the above-mentioned literature, this paper investigates the problem of disturbance attenuation and rejection for switched nonlinear systems with actuator saturation and multiple disturbances. The main contribution and innovation are included as: Firstly, a disturbance observer is designed to estimate the matched disturbance, based on which a novel composite controller is further constructed. It need to notice that the observer and controller are depended on a proposed state-dependent switching law. Secondly, by the multiple Lyapunov function technology, the design conditions of the controller ensuring the robust local asymptotic stability of the closed-loop system without taking account of external disturbances are established. The domain of attraction is estimated and optimized. Thirdly, an algorithm for the largest disturbance is proposed such that the H_∞ performance can be further achieved for the closed-loop system subject to the external disturbances.

II. PRELIMINARIES AND PROBLEM STATEMENT

Consider a class of switched nonlinear systems, the dynamics of which can be expressed as

$$\dot{x}(t) = A_{\varrho(t)}x(t) + B_{\varrho(t)}(u(t) + \omega(t)) + E_{1\varrho(t)}d_1(t) + F_{\varrho(t)}f(x(t)), \quad (1)$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ is the continuous control input. The standard saturation function $: R^m \to R^m$ is defined as:

$$\text{sat}(u) = [\text{sat}(u_1)\ \ \text{sat}(u_2)\ \ \cdots\ \ \text{sat}(u_m)]^T,$$

where $(u_i) = (u_i)\min\{1, |u_i|\}$.

$d_1(t)$ is the external H_2-norm bounded disturbance. the disturbance $\omega(t) \in R^m$ is matched with control inputs, which generated by an exogenous system as follows:

$$\begin{aligned} \dot{\vartheta}(t) &= W_{\varrho(t)}\vartheta(t) + E_{2\varrho(t)}d_2(t), \\ \omega(t) &= V_{\varrho(t)}\vartheta(t). \end{aligned} \quad (2)$$

where $d_2(t)$ is the additional H_2-norm bounded disturbance.

The switched law is described as piecewise constant function $\varrho(t)$ taking values in finite set $I[1, N] = \{1, 2, \cdots N\}$, and the i_{th} subsystem is activated when $\varrho(t) = i \in I[1, N]$.

Remark 1: For $i \in I[1, N]$, the system (2) can generate various disturbance signals by choosing different system matrices W_i and V_i, such as constant disturbance, harmonics and so on [35] [36]. This paper describes the exogenous disturbance system as the switching system (2), the reason of which is that the disturbance should be different for each of subsystems and dependent on the switching law in engineering.

The function $f(x(t))$ satisfying the following assumption is used to model the nonlinear dynamics of the system.

Assumption 1: The nonlinear function $f(x(t))$ satisfies

$$\| f(x_1) - f(x_2) \| \leq \| U(x_1 - x_2) \|, \quad \forall x_1, x_2 \in R^n. \quad (3)$$

where U is a given constant weighting matrix.

In what follows, the following assumption on the system (1) and the disturbance dynamic system (2) is made

Assumption 2: For $\varrho(t) = i$, (A_i, B_i) is controllable and (W_i, V_i) is observable.

By introducing the nonlinear function $\theta(\upsilon) = \upsilon - sat(\upsilon)$ and disturbance dynamic (2), system (1) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= A_{\varrho(t)}x(t) + B_{\varrho(t)}u(t) + B_{\varrho(t)}V_{\varrho(t)}\vartheta(t) \\ &\quad + E_{\varrho(t)}d_1(t) + F_{\varrho(t)}f(x(t)) \\ &\quad - B_{\varrho(t)}\theta(u(t) + V_{\varrho(t)}\vartheta(t)), \end{aligned} \quad (4)$$

In order to derive the results of this paper, the following lemma is referred as:

Lemma 1 [19]: Consider the function $\theta(\upsilon)$ defined above. If $x \in L(K, H)$, then the relation

$$\theta^T(Kx)T[\theta(Kx) - Hx] \leq 0, \tag{5}$$

holds for any diagonal and positive definite matrix $T \in R^{m\times m}$, where

$$L(K, H) = \{x \in R^n : |(K^l - H^l)x| \leq 1, l \in I[1, m]\},$$

with H^l is the lth row of the matrix $H \in m \times n$.

Lemma 2 [33]: Let D, S and F be real matrices of appropriate dimensions with $F^TF \leq I$. Then, for any scalar $\lambda > 0$, we have

$$DFS + (DFS)^T \leq \lambda^{-1}DD^T + \lambda S^TS. \tag{6}$$

Lemma 3 [34]: For $\forall x, y \in R^n$ and any positive definite matrix $P \in R^{n\times n}$, we have

$$2x^Ty \leq x^TPx + y^TP^{-1}y. \tag{7}$$

In what follows, we suppose that $f(x(t))$ is given which satisfies Assumption 1 and all states of the system (1) are available. Under Assumption 2, a switching disturbance observer is constructed to estimate the exosystem generated disturbance $\omega(t)$ as follows:

$$\begin{aligned}
\hat{\omega}(t) &= V_{\varrho(t)}\hat{\vartheta}(t),\\
\hat{\vartheta}(t) &= \varphi(t) - Lx(t),\\
\dot{\varphi}(t) &= (W_{\varrho(t)} + LB_{\varrho(t)}V_{\varrho(t)})(\varphi(t) - Lx(t))\\
&\quad + L[A_{\varrho(t)}x(t) + B_{\varrho(t)}u(t) + F_{\varrho(t)}f(x(t))].
\end{aligned} \tag{8}$$

Based on (8), a composite controller can be formulated as

$$u = K_{\varrho(t)}x(t) - \hat{\omega}(t) \tag{9}$$

After defining the error $e_\vartheta(t) = \vartheta(t) - \hat{\vartheta}(t)$, an augmented closed-loop system can be obtained from (4), (8) and (9) as

$$\dot{\eta}(t) = \tilde{A}_{\varrho(t)}\eta(t) + \tilde{F}_{\varrho(t)}f(\eta(t)) + \tilde{B}_{\varrho(t)}\theta(\tilde{K}_{\varrho(t)}\eta(t)) + \tilde{E}_{\varrho(t)}d(t) \tag{10}$$

where $\eta(t) = [x^T(t)\ e_\vartheta^T(t)]^T$, $f(\eta(t)) = f(x(t))$ and

$$\begin{aligned}
\tilde{A}_{\varrho(t)} &= \begin{bmatrix} A_{\varrho(t)} + B_{\varrho(t)}K_{\varrho(t)} & B_{\varrho(t)}V_{\varrho(t)} \\ 0 & W_{\varrho(t)} + LB_{\varrho(t)}V_{\varrho(t)} \end{bmatrix},\\
\tilde{B}_{\varrho(t)} &= \begin{bmatrix} -B_{\varrho(t)} \\ -LB_{\varrho(t)} \end{bmatrix}, \quad \tilde{E}_{\varrho(t)} = \begin{bmatrix} E_{1\varrho(t)} & 0 \\ LE_{1\varrho(t)} & E_{2\varrho(t)} \end{bmatrix},\\
\tilde{F}_{\varrho(t)} &= \begin{bmatrix} F_{\varrho(t)} \\ 0 \end{bmatrix}, \quad \tilde{K}_{\varrho(t)} = \begin{bmatrix} K_{\varrho(t)} & V_{\varrho(t)} \end{bmatrix},
\end{aligned}$$

Assumption 3: In this paper, the energy of the disturbances $d_1(t)$ and $d_2(t)$ is bounded by a given value, i.e.,

$$\Pi_\alpha = \left\{d_i(t) : \int_0^\infty d_i^T(t)d_i(t)dt \leq \frac{\alpha}{2}, i = 1, 2,\right\} \tag{11}$$

for some $\alpha > 0$.

The objective of this paper is to design a switching law and obtain some design conditions of the composite controller (9) based on the disturbance observer (8), such that the disturbances can be rejected and attenuated under the guarantee of stability of the argument system (10).

III. MAIN RESULTS

Before presenting the main results, an abbreviation need to be made as

$$\Omega(P, \varrho) = \{x \in R^p : x^TPx \leq \varrho\}, \tag{12}$$

where $\varrho \in R$.

In this section, we will establish two theorems to achieve the required control performance along with disturbance attenuation and rejection for the closed-loop system (10).

Theorem 1: Under the assumption $d(t) \equiv 0$, if there exist matrices $X_{1i} > 0$, $P_2 > 0$, Y, U_i, Z_i, H_{2i}, diagonal matrices $Q_i > 0$ and scalars $\delta_{ir} > 0$, $i, r \in I[1, N]$, such that

$$\begin{bmatrix} 1 & U_i^l - Z_i^l & V_i^l - H_{2i}^l & 0 \\ * & (1 + \sum_{r\neq i}\delta_{ir})X_{1i} & 0 & \varpi_i^3 \\ * & * & P_2 & 0 \\ * & * & * & \varpi_i^4 \end{bmatrix} > 0, \tag{13}$$

and (14),

$$\begin{bmatrix} \varpi_i^1 & B_iV_i & -B_iQ_i + Z_i^T & X_{1i}U^T & \varpi_i^3 \\ * & \varpi_i^2 & -YB_iQ_i + H_{2i}^T & 0 & 0 \\ * & * & -2Q_i & 0 & 0 \\ * & * & * & -\varepsilon_{1i}I & 0 \\ * & * & * & * & -\varpi_i^4 \end{bmatrix} < 0, \tag{14}$$

where Z_i^l, H_{2i}^l are the l_{th} row of Z_i, H_{2i} respectively and

$$\begin{aligned}
\varpi_i^1 &= He\{A_i + B_iU_i\} + \varepsilon_{1i}F_iF_i^T - \sum_{r\neq i}\delta_{ir}X_{1i},\\
\varpi_i^2 &= He\{P_2W_i + YB_iV_i\},\\
\varpi_i^3 &= [\sqrt{\delta_{11}}\cdots\sqrt{\delta_{1i-1}}\ \sqrt{\delta_{1i+1}}\cdots\sqrt{\delta_{1N}}]X_{1i},\\
\varpi_i^4 &= X_{11}, \cdots, X_{1i-1}, X_{1i+1}, \cdots, X_{1m}.
\end{aligned}$$

then, under the switching law

$$\varrho = arg\min\{\eta^T\bar{P}_i\eta, i \in I_N\}, \tag{15}$$

system (10) is locally asymptotically stable with an estimation of the domain of attraction as

$$\cup_{i=1}^N(\Omega(\bar{P}_i, 1) \cap \varsigma_i), \tag{16}$$

where $\varsigma_i = \{\eta(t) : \eta^T(\bar{P}_r - \bar{P}_i)\eta \geq 0, \forall r \in I_N, r \neq i\}$, $\bar{P}_i = dig\{P_{1i}, P_2\}$ and $\bar{H}_i = [H_{1i}\ H_{2i}]$ with $P_{1i} = X_{1i}^{-1}$ and $H_{1i} = Z_iP_{1i}$. The control law (9) based on (8) can be designed as $K_{1i} = U_iP_{1i}$ and $L = P_2^{-1}Y$.

Proof: Choose the multiple Lyapunov function as

$$V(\eta(t)) = \eta^T(t)\bar{P}_{\varrho(t)}\eta(t) \tag{17}$$

For $\varrho(t) = i \in I[1, N]$, under the assumption of $d(t) = 0$, computing the time derivative of $V(t)$ in (17) yields

$$\begin{aligned}
\dot{V}(\eta(t)) &= 2\eta^T(t)\bar{P}_i\dot{\eta}(t)\\
&= \eta^T(t)(\bar{P}_i\tilde{A}_i + \tilde{A}_i\bar{P}_i)\eta(t) + 2\eta^T(t)\bar{P}_i\\
&\quad \times \tilde{F}_if(\eta(t)) + 2\eta^T(t)\bar{P}_i\tilde{B}_i\theta(\tilde{K}_i\eta(t)).
\end{aligned} \tag{18}$$

Under Assumption 1, using Lemma 2 to (18) leads to

$$\dot{V}(\eta(t)) \leq \eta^T(t)(\bar{P}_i\tilde{A}_i + \tilde{A}_i\bar{P}_i)\eta(t) + 2\eta^T(t)\bar{P}_i\tilde{B}_i\theta(\bar{K}_i\eta(t)) + \eta^T(t)[\varepsilon_{1i}^{-1}\tilde{U}_i^T\tilde{U}_i + \varepsilon_{1i}\bar{P}_i\tilde{F}_i\tilde{F}_i^T\bar{P}_i]\eta(t), \quad (19)$$

where ε_{1i} is any positive scalar and $\tilde{U}_i = [U_i\ 0]$ with a given matrix U_i. By choosing $P_{1i} = X_{1i}^{-1}, H_i = P_{1i}Z_i, K_{1i} = U_{1i}P_{1i}$ and using the Schur complement, (13) can be rewritten as:

$$\begin{bmatrix} 1 & K_{1i}^l - H_{1i}^l & V_i^l - H_{2i}^l \\ * & P_{1i} + \sum_{r\neq i}\delta_{ir}(P_{1i} - P_{1r}) & 0 \\ * & * & P_2 \end{bmatrix} > 0, \quad (20)$$

which is equivalent to

$$(\tilde{K}_i^l - \bar{H}_i^l)(\bar{P}_i - \sum_{r\neq i}\delta_{ir}(\bar{P}_r - \bar{P}_i))^{-1}(\tilde{K}_i^l - \bar{H}_i^l)^T < 1. \quad (21)$$

On the hand, for $\eta(t) \in \Omega(\bar{P}_i, 1) \cap \varsigma_i$, one has

$$\eta^T(t)(\bar{P}_i - \sum_{r\neq i}\delta_{ir}(\bar{P}_r - \bar{P}_i))\eta(t) < 1. \quad (22)$$

Resorting to Lemma 3 with (21) and (22), it follows $\eta(t) \in L(\tilde{K}_i, \bar{H}_i)$.

For $\eta(t) \in \Omega(\bar{P}_i, 1) \cap \varsigma_i \subset L(\tilde{K}_i, \bar{H}_i)$, applying Lemma 1 and recalling (19) yield

$$\dot{V}(t) \leq \eta^T(t)(\bar{P}_i\tilde{A}_i + \tilde{A}_i\bar{P}_i + \varepsilon_{1i}^{-1}\tilde{U}^T\tilde{U} + \varepsilon_{1i}\bar{P}_i\tilde{F}_i\tilde{F}_i^T\bar{P}_i)\eta(t) + 2\eta^T(t)\bar{P}_i\tilde{B}_i\theta(\tilde{K}_i\eta(t)) - 2\theta^T(\tilde{K}_i\eta(t))T_i[\theta(\tilde{K}_i\eta(t)) - \bar{H}_i\eta(t)], \quad (23)$$

where $\tilde{U} = [U^T\ 0^T]^T$.

Based on the Schur complement and matrix congruence transformation, (13) can be rewritten as

$$\begin{bmatrix} \pi_{1i}^{11} & \bar{P}_i\tilde{B}_i + \bar{H}_iT_i \\ * & -2T_i \end{bmatrix} < 0, \quad (24)$$

where

$$T_i = Q_i^{-1},$$
$$\pi_{1i}^{11} = He\{\bar{P}_i\tilde{A}_i\} + \varepsilon_{1i}^{-1}\tilde{U}^T\tilde{U} + \varepsilon_{1i}\bar{P}_i\tilde{F}_i\tilde{F}_i^T\bar{P}_i + \sum_{r\neq i}\delta_{ir}(\bar{P}_r - \bar{P}_i).$$

For any $\mu(t) = [\eta^T(t)\ \theta^T(\tilde{K}_i\eta(t))]^T \neq 0$, it is the fact that

$$\eta^T(t)(\bar{P}_i\tilde{A}_i + \tilde{A}_i\bar{P}_i + \varepsilon_{1i}^{-1}\tilde{U}^T\tilde{U} + \varepsilon_{1i}\bar{P}_i\tilde{F}_i\tilde{F}_i^T\bar{P}_i)\eta(t) + 2\eta^T(t)\bar{P}_i\tilde{B}_i\theta(\tilde{K}_i\eta(t)) - 2\theta^T(\tilde{K}_i\eta(t))T_i[\theta(\tilde{K}_i\eta(t)) - \bar{H}_i\eta(t)] + \sum_{r\neq i}\delta_{ir}[\eta^T(t)(\bar{P}_r - \bar{P}_i)\eta(t)] < 0. \quad (25)$$

Under switching law (15), when $\eta(t) \in \Omega(\bar{P}_i, 1) \cap \varsigma_i \subset L(\tilde{K}_i, \bar{H}_i)$, one has $\varrho(t) = i$ and

$$\eta^T(t)(\bar{P}_r - \bar{P}_i)\eta(t) > 0, \quad r \neq i. \quad (26)$$

Combining (23), (25) and (26) yields that $\dot{V}(\eta(t)) < 0$ for any $\eta(t) \in \Omega(\bar{P}_i, 1) \cap \varsigma_i \subset L(\tilde{K}_i, \bar{H}_i)$.

At a switching instant $t = t_k$, under switching law (15), we have $V(\eta(t_k)) = \eta^T(t_k)\bar{P}_{\varrho(t_k)}\eta(t_k) < \eta^T(t_k)\bar{P}_{\varrho(t_k^-)}\eta(t_k) = \eta^T(t_k^-)\bar{P}_{\varrho(t_k)}\eta(t_k^-) = V(\eta(t_k^-))$.

For any $t \in [t_k, t_{k+1})$, integrating $\dot{V}(t) < 0$ from 0 to t results in

$$\begin{aligned} 0 &> \int_0^t \dot{V}(\tau)d\tau \\ &= \int_0^{t_1}\dot{V}(\tau)d\tau + \int_{t_1}^{t_2}\dot{V}(\tau)d\tau + \cdots + \int_{t_j}^{t_k}\dot{V}(\tau)d\tau + \int_{t_k}^{t}\dot{V}(\tau)d\tau \\ &= V(\eta(t_1^-)) - V(\eta(0)) + V(\eta(t_2^-)) - V(\eta(t_1)) + \cdots + V(\eta(t_k^-)) - V(\eta(t_j)) + V(\eta(t)) - V(\eta(t_k)) \\ &= V(\eta(t)) - V(\eta(0)) + (V(\eta(t_1^-)) - V(\eta(t_1))) + \cdots + (V(\eta(t_k^-)) - V(\eta(t_k))) > V(\eta(t)) - V(\eta(0)) \end{aligned} \quad (27)$$

It follows $V(\eta(t)) < V(\eta(0))$, which implies that $\eta(t) \in \Omega(\bar{P}_{\varrho_{t_k}}) \cap \varsigma_{\varrho_{t_k}} \subset \cup_{i=1}^N(\Omega(\bar{P}_i, 1) \cap \varsigma_i)$ if $\eta(0) \in \cup_{i=1}^N(\Omega(\bar{P}_i, 1) \cap \varsigma_i)$.

Consequently, system (10) under composite controller (9) and switching law (15) is locally asymptotically stable with the estimation of the domain of attraction (16). This completes the proof. ■

Remark 2: In Theorem 1, we have designed a state-dependent switching law and a switching DOB controller, under which system (10) with $d(t) = 0$ can be local asymptotically stable, even though all of subsystems of (10) are unstable. It means that the estimation of the exosystem generated disturbance $\omega(t)$ and the stabilization of system (1) cannot be implemented effectively without switching law (15).

Remark 3: The conditions in Theorem 1 are sufficient for the local stability of system (10). Therefore, it is important to calculate the gains of controller (9) and disturbance observer (8) based on (13) and (14) such that the estimation of the domain of attraction is as large as possible.

Based on the discussion in Remark 3, the following iterative algorithm is proposed:

Remark 4: For the implementation of Algorithm 1, the initialization of matrix Y can be set based on the solution of the following LMIs:

$$He\{PW_i + YB_iV_i\} < 0, \quad i \in I[1, N],$$

which are the conditions of the stability of the stability of the disturbance dynamic error system: $\dot{e}_\vartheta(t) = (W_{\varrho(t)} + LB_{\varrho(t)}V_{\varrho(t)})\vartheta(t)$ under the arbitrary switching signal with $L = P^{-1}Y$.

In this paper, the reference output is set to be

$$\tilde{z}(t) = \tilde{C}_i\eta(t), \quad (28)$$

where $\tilde{C}_i = [C_{1i}\ 0]$.

The augmented closed-loop system (10) is called to have a restricted L_2-gain less than or equal to γ, if under zero initial

Algorithm 1 The Optimizing Algorithm of Estimation of the Domain of Attraction

Step 1: Choose the appropriate matrices R, Y and scalars $\delta_{ir} > 0$.

Step 2: Solve linear matrix inequalities (LMIs) (13) and (14) for Q_i.

Step 3: For obtained Q_i in Step 2, Solve the following problem for Y and μ:

$$\inf_{X_{1i}>0,P_2>0,U_i,Z_i,H_{2i}} \mu$$
$$s.t. \quad a) \begin{bmatrix} \mu R & 0 & I \\ * & \mu R - P_2 & 0 \\ * & * & X_{1i} \end{bmatrix} \geq 0,$$
$b)$ inequalities (13) and (14) hold.

Step 4: If $|\mu_{\text{new}} - \mu_{\text{old}}| < \epsilon$, where $\epsilon > 0$ is a fixed small number, stop the iteration; otherwise, go back to Step 2 with Y obtained from Step 3.

condition, the following performance inequality holds

$$\|\tilde{z}(t)\|_2 < \gamma\|d(t)\|_2, \quad \forall: \ d(t) \in \Pi_\alpha.$$

Theorem 2: If for prescribed scalars $\gamma > 0$, there exist $X_{1i} > 0, P_2 > 0, Y, U_i, Z_i, H_{2i}$, diagonal matrices $Q_i > 0$ and scalars $\varepsilon_{1i} > 0, \delta_{ir} >, i, r \in I[1, N]$, such that

$$\begin{bmatrix} 1 & U_i^l - Z_i^l & dV_i^l - H_{2i}^l & 0 \\ * & \frac{1}{\alpha}(1 + \sum_{r\neq i}\delta_{ir})X_{1i} & 0 & \pi_i^3 \\ * & * & P_2 & 0 \\ * & * & * & \varpi_i^4 \end{bmatrix} > 0, \tag{29}$$

and (30) in the top of next page with Z_i^l, H_{2i}^l, ϖ_i^2, ϖ_i^3, ϖ_i^4 defined in Theorem 1, and

$$\pi_i^1 = He\{A_i + B_iU_i\} + \varepsilon_{1i}F_iF_i^T - \sum_{r\neq i}\delta_{ir}X_{1i} + E_{1i}E_{1i}^T,$$
$$\pi_i^3 = [\sqrt{\frac{\delta_{11}}{\alpha}} \cdots \sqrt{\frac{\delta_{1i-1}}{\alpha}} \ \sqrt{\frac{\delta_{1i+1}}{\alpha}} \cdots \sqrt{\frac{\delta_{1N}}{\alpha}}]X_{1i},$$

then, under the switching law (15), the restricted L_2 gain from $d(t)$ to $\tilde{z}(t)$ for system (10) is less than or equal to γ within $\cup_{i=1}^N(\Omega(\bar{P}_i, \alpha) \cap \varsigma_i)$, the control law (9) based on (8) and H_∞ controller can be designed as $K_{1i} = U_iP_{1i}$, $L = P_2^{-1}Y$ and $H_i = Z_iP_{1i}$ with $P_{1i} = X_{1i}^{-1}$.

Proof: Setting $P_{1i} = X_{1i}^{-1}, Q_i = T_i^{-1}, K_{1i} = U_iP_{1i}, L = P_2^{-1}Y$ and $H_i = P_{1i}Z_i$ and applying the Schur complement for inequalities (29) yields that

$$(\tilde{K}_i^l - \bar{H}_i^l)[\frac{1}{\alpha}(\bar{P}_i - \sum_{r\neq i}\delta_{ir}(\bar{P}_r - \bar{P}_i))]^{-1}(\tilde{K}_i^l - \bar{H}_i^l)^T < 1. \tag{31}$$

Under the switching law (15), for $\varrho(t) = i \in I[1, N]$ and $\eta(t) \in \Omega(\bar{P}_i, \alpha) \cap \varsigma_i$, we have

$$\eta^T(t)(\bar{P}_i - \sum_{r\neq i}\delta_{ir}(\bar{P}_r - \bar{P}_i))\eta(t) < \alpha, \tag{32}$$

which can be rewritten as

$$\eta^T(t)[\frac{1}{\alpha}(\bar{P}_i - \sum_{r\neq i}\delta_{ir}(\bar{P}_r - \bar{P}_i))]\eta(t) < 1. \tag{33}$$

By Lemma 3, it then follows from (31) and (34) that

$$2(\tilde{K}_i^l - \bar{H}_i^l)\eta(t) < 2, \tag{34}$$

which implies that $\eta(t) \in \Omega(\bar{P}_i, \alpha) \cap \varsigma_i \subset L(\tilde{K}_i, \ \bar{H}_i)$.

Define a multiple Lypunonv function as

$$V(\eta(t)) = \eta^T(t)\bar{P}_{\varrho(t)}\eta(t) \tag{35}$$

where $\bar{P}_{\varrho(t)} = P_{1\varrho(t)}, P_2$.

For $t \in [t_k, t_{k+1})$ with switching instants t_k and t_{k+1}, it is assumed that $\varrho(t) = i \in I[1, N]$. By Assumption 1, Lemma 1 and Lemma 2, for $\eta(t) \in \Omega(\bar{P}_i, \alpha) \cap \varsigma_i$, we have

$$\begin{aligned} \dot{V}(t) \leq & \ \eta^T(t)(\bar{P}_i\tilde{A}_i + \tilde{A}_i\bar{P}_i + \varepsilon_{1i}^{-1}\tilde{U}^T\tilde{U} \\ & + \varepsilon_{1i}\bar{P}_i\bar{F}_i\bar{F}_i^T\bar{P}_i + \sum_{r\neq i}\delta_{ir}(\bar{P}_r - \bar{P}_i))\eta(t) \\ & + \eta^T(t)\bar{P}_i\bar{E}_i\bar{E}_i^T\bar{P}_i\eta(t) + d^T(t)d(t) \\ & + 2\eta^T(t)\bar{P}_i\tilde{B}_i\theta(\tilde{K}_i\eta(t)) \\ & - 2\theta^T(\tilde{K}_i\eta(t))T_i[\theta(\tilde{K}_i\eta(t)) - \bar{H}_i\eta(t)] \\ = & \ \mu^T(t)\Pi_{1i}\mu(t) + d^T(t)d(t) \end{aligned} \tag{36}$$

where

$$\Pi_{1i} = \begin{bmatrix} \pi_{2i}^{11} & \bar{P}_i\tilde{B}_i + \bar{H}_i^T T_i \\ * & -2T_i \end{bmatrix},$$

with

$$\begin{aligned} \pi_{2i}^{11} = & \ He\{\bar{P}_i\tilde{A}_i\} + \varepsilon_{1i}^{-1}\tilde{U}^T\tilde{U} + \varepsilon_{1i}\bar{P}_i\bar{F}_i\bar{F}_i^T\bar{P}_i \\ & + \sum_{r\neq i}\delta_{ir}(\bar{P}_r - \bar{P}_i) + \bar{P}_i\bar{E}_i\bar{E}_i^T\bar{P}_i. \end{aligned}$$

Pre- and post- multiplying (30) by $\{P_{1i}, I, T_i, I\}$ and using the Schur complement for (30) yields that

$$\Pi_i = \begin{bmatrix} \pi_{2i}^{11} + \frac{1}{\gamma^2}\tilde{C}_i^T\tilde{C}_i & \bar{P}_i\tilde{B}_i + \bar{H}_i^T T_i \\ * & -2T_i \end{bmatrix} < 0. \tag{37}$$

Firstly, (36) and (37) imply that

$$\dot{V}(t) \leq d^T(t)d(t), \quad t \in [t_k, t_{k+1}). \tag{38}$$

Under the switching law (15) and the assumption of $\eta(0) = 0$, integrating both sides of the above inequality from 0 to t results in

$$\begin{aligned} \alpha > & \int_0^t d^T(\tau)d(\tau)d\tau > \int_0^t \dot{V}(\tau)d\tau \\ & = \int_0^{t_1} \dot{V}(\tau)d\tau + \int_{t_1}^{t_2} \dot{V}(\tau)d\tau \end{aligned}$$

$$\begin{bmatrix} \pi_i^1 & B_iV_i + E_{1i}E_{1i}^TY^T & -B_iQ_i + Z_i^T & X_{1i}U^T & \varpi_i^3 & X_{1i}C_{1i}^T & 0 \\ * & \varpi_i^2 & -YB_iQ_i + H_{2i}^T & 0 & 0 & 0 & P_2E_{2i} \\ * & * & -2Q_i & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon_{1i}I & 0 & 0 & 0 \\ * & * & * & * & \varpi_i^4 & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{30}$$

$$\begin{aligned} &+ \cdots + \int_{t_j}^{t_k} \dot{V}(\tau)d\tau + \int_{t_k}^{t} \dot{V}(\tau)d\tau \\ &= V(\eta(t_1^-)) - V(\eta(0)) + V(\eta(t_2^-)) \\ &\quad - V(\eta(t_1)) + \cdots + V(\eta(t_k^-)) \\ &\quad - V(\eta(t_j)) + V(\eta(t)) - V(\eta(t_k)) \\ &= V(\eta(t)), \end{aligned} \tag{39}$$

which means every trajectory of system (10) that starts from a zero initial condition remains insides $\cup_{i=1}^N(\Omega(\bar{P}_i, \alpha) \cap \varsigma_i)$ for every $d(t) \in \Pi_\alpha$.

On the other hand, from (36) and (37), it follows that

$$\begin{aligned} \dot{V}(t) \leq\ & \mu^T(t)\Pi_{2i}\mu(t) - \frac{1}{\gamma^2}\eta^T(t)\tilde{C}_i^T\tilde{C}_i\eta(t) + d^T(t)d(t) \\ \leq\ & -\frac{1}{\gamma^2}\eta^T(t)\tilde{C}_i^T\tilde{C}_i\eta(t) + d^T(t)d(t) \\ =\ & -\frac{1}{\gamma^2}z^T(t)z(t) + d^T(t)d(t). \end{aligned} \tag{40}$$

Similarly, we have

$$0 < V(t) \leq -\frac{1}{\gamma^2}\int_0^t z^T(\tau)z(\tau)d\tau + \int_0^t d^T(\tau)d(\tau)d\tau, \tag{41}$$

It is obvious that (41) is equivalent to $\|\tilde{z}(t)\|_2 < \gamma\|d(t)\|_2$, which implies that system (10) has a restricted L_2 gain less than γ. ■

Remark 5: In Theorem 2, the conditions are presented to guarantee the system (10) has a restricted L_2 gain less than or equal to γ for any $d(t) \in \Pi_\alpha$. Therefore, it is important to optimize the value of α to describe the largest disturbances can be tolerated by the system (10) and estimate the set $\cup_{i=1}^N(\Omega(\bar{P}_i, \alpha)\cap\varsigma_i)$, in which the state trajectories with zero initial condition are bounded.

In what follows, we will give an algorithm for the largest α such that the results in Theorem 2 hold.

Algorithm 2 The Optimizing Algorithm for Largest Disturbance

Step 1: Choose the appropriate matrices $Q_i > 0$ and scalars $\delta_{ir} > 0$.

Step 2: Solve LMIs (29) and (30) for Y.

Step 3: For obtained Y in Step 2, Solve the following problem for Q_i and α:

$$\sup_{X_{1i}>0, P_2>0, U_i, Z_i, H_{2i}} \alpha$$
$$s.t. \quad \text{inequalities (29) and (30) hold.}$$

Step 4: If $|\alpha_{\text{new}} - \alpha_{\text{old}}| < \epsilon$, where $\epsilon > 0$ is a fixed small number, stop the iteration; otherwise, go back to Step 2 with Q_i obtained from Step 3.

IV. NUMERICAL EXAMPLE

Let consider systems (1) and (2) including two subsystems with the parameters as follows:

Mode 1:

$$A_1 = \begin{bmatrix} 0.1 & 0.5 \\ 0.1 & -0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.1 \\ 0.3 \end{bmatrix}, F_1 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},$$
$$W_1 = \begin{bmatrix} 3 & 0 \\ 0 & -3 \end{bmatrix}, \quad E_{11} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, E_{21} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$
$$V_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}^T, \quad C_{11} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}^T;$$

Mode 2:

$$A_2 = \begin{bmatrix} -0.3 & 0.05 \\ -0.1 & 0.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, F_2 = \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix},$$
$$W_2 = \begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}, E_{12} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}, E_{22} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix},$$
$$V_2 = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}^T, \quad C_{12} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}^T.$$

With the above parameters, we specify initial conditions of the Algorithm **1** by $R = Q_1 = Q_2 = I$, $\varepsilon_{11} = 0.1$, $\varepsilon_{12} = 0.5$, $\delta_{12} = 0.5$ and $\delta_{21} = 1$, the gains of the composite controller (9) under the optimized parameter $\mu = 0.7362$ are obtained as follows:

$$\begin{aligned} L &= \begin{bmatrix} 5.0994 & -15.7605 \\ 5.1067 & -15.7830 \end{bmatrix}, \\ K_1 &= \begin{bmatrix} -1.2372 & -2.2427 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} -0.3321 & -1.5341 \end{bmatrix}. \end{aligned} \tag{42}$$

We conduct the simulation to verify our composite control deign with the above parameters and the initial conditions which are set as $x(0) = [-0.3\ \ -0.1]^T$ and $e_\vartheta = [0.1\ 0.1]^T$.

Firstly, by supposing $d_1(t) = d_2(t) = 0$ and $f(x) = x_2\, e^{-\frac{t}{10}}$, under the switching law (15) and gains in (43), it can be seen from Figs. 1–2 that the disturbance observer (8) can estimate the exosystem generated disturbance $\omega(t)$ effectively and the states of the closed-loop system are asymptotically

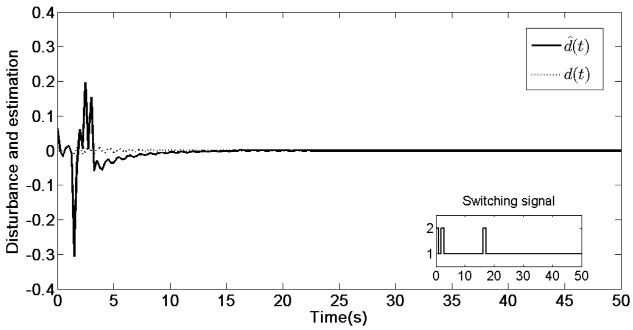

FIGURE 1. **Curves of disturbance and disturbance estimation in the case of $d_1(t) = d_2(t) = 0$.**

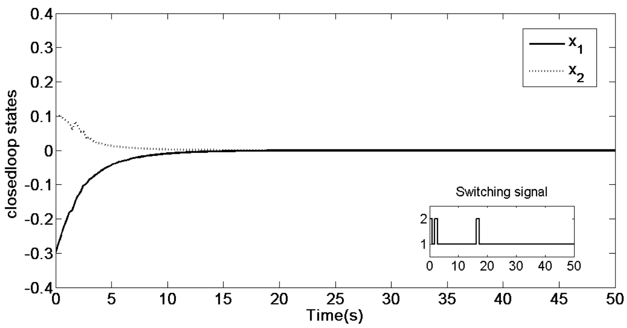

FIGURE 2. **Response curves of the closed-loop system state in the case of $d_1(t) = d_2(t) = 0$.**

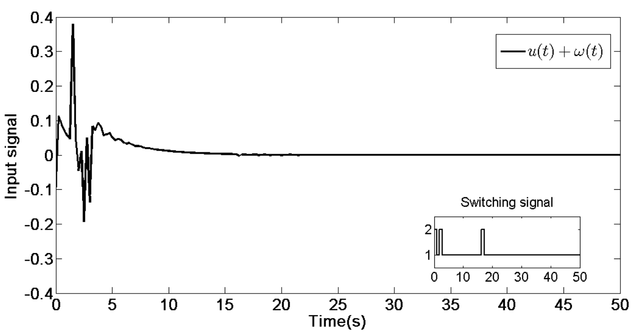

FIGURE 3. **Curve of input signal $u(t) + \omega(t)$ in the case of $d_1(t) = d_2(t) = 0$.**

convergent to zero. Moreover, the input signal of the system including controlled and disturbance inputs is depicted in Fig. 3.

Next, by solving the Algorithm 2 with the initial parameters $Q_1 = Q_2 = R$, $\varepsilon_{11} = 0.1$, $\varepsilon_{12} = 0.5$, $\delta_{12} = 0.5$ and $\delta_{21} = 1$, a feasible solution of optimized parameters is $\alpha = 42.6732$, and the gains of the controller (9) can be further presented as

$$L = \begin{bmatrix} -8.5421 & -44.1503 \\ -8.5496 & -44.1871 \end{bmatrix},$$
$$K_1 = \begin{bmatrix} -5.3883 & -6.1112 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -2.9402 & -6.2276 \end{bmatrix}. \quad (43)$$

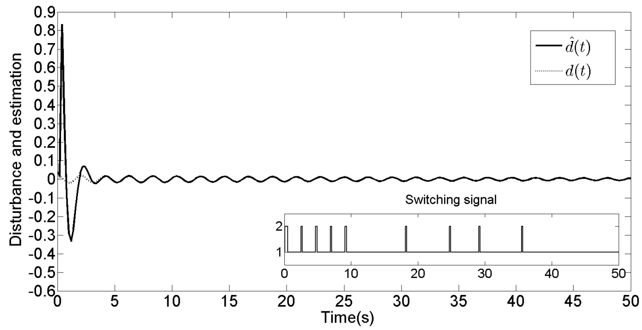

FIGURE 4. **Curves of disturbance and disturbance estimation.**

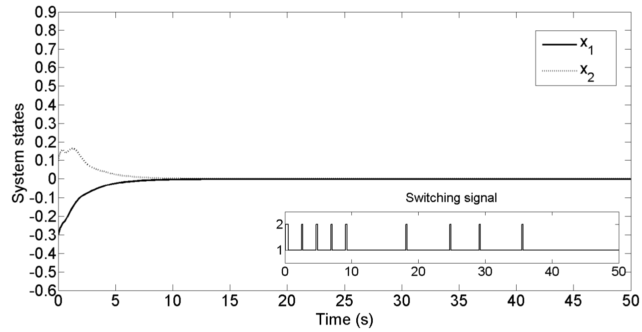

FIGURE 5. **Response curves of the closed-loop system states.**

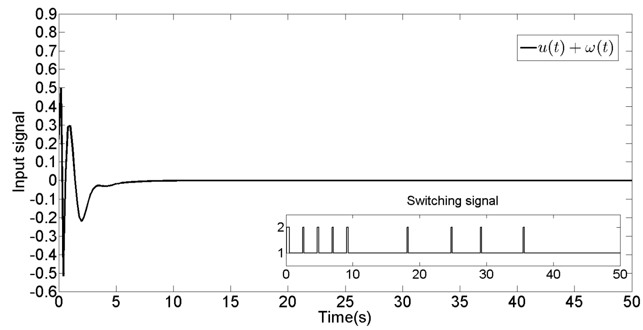

FIGURE 6. **Curves of input signal $u(t) + \omega(t)$.**

We suppose external disturbances $d_1(t) = e^{-2t}$ and $d_2(t) = e^{-t}$, the results in Theorem 2 are be confirmed in the following simulation. Fig. 4 shows the trajectories of both the disturbances $\omega(t)$ and the corresponding estimation $\hat{\omega}(t)$ based on the observer (9), which can demonstrate the effectiveness of the disturbance observer. The curves of the closed-loop system states are described in Fig. 5, which shows that the system under multiple disturbances (1) can be controlled by the designed controller (10). In Figs. 6-7, the control input and output are described, respectively. It need to be mentioned that the results of the simulations in Figs. 4-7 are obtained under the switching law (15).

Remark 6: In [37], the robust stabilization of switched systems with nonlinear uncertainty and actuator saturation has been investigated. For the switched system under consideration in this example, the gains of the controller in [37] can

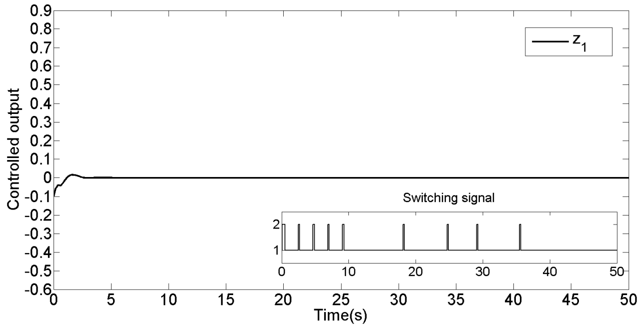

FIGURE 7. Curves of output signal $z(t)$.

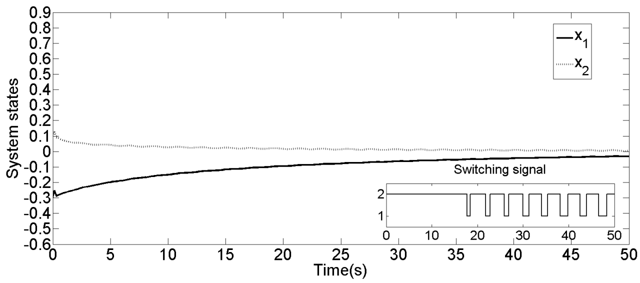

FIGURE 8. Response curves of the closed-loop system state under the control scheme in [37] with disturbances $\omega(t)$, $d_1(t)$ and $d_2(t)$.

be solved as

$$K_1 = \begin{bmatrix} -1.4290 & -3.6721 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -5.6722 & -2.1417 \end{bmatrix}. \tag{44}$$

However, Fig. 8 shows that the capabilities of this controller deteriorate seriously, when the switched system is subject to both exosystem generated disturbance and H_∞-norm bounded disturbance. By the proposed control scheme in this paper, the control performance of the system can be achieved effectively from Fig. 5.

V. CONCLUSION

The problem of composite anti-disturbance control for a class of switched nonlinear systems subject to input saturation and multiple disturbances has been investigated. Based on the switching signal, the novel disturbance observer and composite controller have been proposed. By designing the state-dependent switching law, some sufficient design conditions for disturbance observer and switching based controller have been given for ensuring the locally asymptotically stable of the closed-loop system and achieving the H_∞ performance requirement. The domain of attraction of the stable closed-loop system has been estimated and optimized, and an algorithm for the largest disturbance has been proposed. Finally, a simulation example is given to show the effectiveness of the results in this paper.

REFERENCES

[1] P. Niamsup and V. N. Phat, "Robust finite-time H_∞ control of linear time-varying delay systems with bounded control via Riccati equations," *IEEE Trans. Autom. Control*, vol. 15, no. 3, pp. 355–363, Jun. 2018.

[2] P. A. Ioannou, J. Sun, *Robust Adaptive Control*. Upper Saddle River, NJ, USA: Prentice-Hall, 1996.

[3] Y. Xia and Y. Jia, "Robust sliding-mode control for uncertain time-delay systems: An LMI approach," *IEEE Trans. Autom. Control*, vol. 48, no. 6, pp. 1086–1091, Jun. 2003.

[4] D. Yue, Q. L. Han, and J. Lam, "Network-based robust H_∞ control of systems with uncertainty," *Automatica*, vol. 41, no. 6, pp. 999–1007, Jun. 2005.

[5] L. Guo and W.-H. Chen, "Disturbance attenuation and rejection for systems with nonlinearity via DOBC approach," *Int. J. Robust Nonlinear Control*, vol. 15, no. 3, pp. 109–125, 2005.

[6] J. Yang, S. Li, and W.-H. Chen, "Nonlinear disturbance observer-based control for multi-input multi-output nonlinear systems subject to mismatching condition," *Int. J. Control*, vol. 85, no. 8, pp. 1071–1082, 2012.

[7] W.-H. Chen, J. Yang, L. Guo, and S. Li, "Disturbance-observer-based control and related methods—An overview," *IEEE Trans. Ind. Electron.*, vol. 63, no. 2, pp. 1083–1095, Feb. 2016.

[8] L. Zhou, Z. Che, and C. Yang, "Disturbance observer-based integral sliding mode control for singularly perturbed systems with mismatched disturbances," *IEEE Access*, vol. 6, pp. 9854–9861, 2018.

[9] L. Guo and S. Cao, *Anti-Disturbance Control for Systems With Multiple Disturbances*. Boca Raton, FL, USA: CRC Press, 2013.

[10] X. Yao and L. Guo, "Composite anti-disturbance control for Markovian jump nonlinear systems via disturbance observer," *Automatica*, vol. 49, no. 8, pp. 2538–2545, 2013.

[11] X. Wei and N. Chen, "Composite hierarchical anti-disturbance control for nonlinear systems with DOBC and fuzzy control," *Int. J. Robust Nonlinear Control*, vol. 24, no. 2, pp. 362–373, 2014.

[12] C. Peng, J. Fang, and S. Xu, "Composite anti-disturbance controller for magnetically suspended control moment gyro subject to mismatched disturbances," *Nonlinear Dyn.*, vol. 79, no. 2, pp. 1563–1573, 2015.

[13] H. Sun, Y. Li, G. Zong, and L. Hou, "Disturbance attenuation and rejection for stochastic Markovian jump system with partially known transition probabilities," *Automatica*, vol. 89, pp. 349–357, Mar. 2018.

[14] A. Aboudonia, R. Rashad, and A. El-Badawy, "Composite hierarchical anti-disturbance control of a quadrotor UAV in the presence of matched and mismatched disturbances," *J. Intell. Robot. Syst.*, vol. 90, nos. 1–2, pp. 201–216, 2018.

[15] T. Hu, Z. Lin, *Control Systems With Actuator Saturation: Analysis and Design*. Springer, 2001.

[16] S. Tarbouriech and M. Turner, "Anti-windup design: An overview of some recent advances and open problems," *IET Control Theory Appl.*, vol. 3, no. 1, pp. 1–19, 2009.

[17] S. Xu, G. Feng, Y. Zou, and J. Huang, "Robust controller design of uncertain discrete time-delay systems with input saturation and disturbances," *IEEE Trans. Autom. Control*, vol. 57, no. 10, pp. 2604–2609, Oct. 2012.

[18] D. Li, Y. Xi, and F. Gao, "Synthesis of dynamic output feedback RMPC with saturated inputs," *Automatica*, vol. 49, no. 4, pp. 949–954, 2013.

[19] S. Tarbouriech *et al.*, *Stability and Stabilization of Linear Systems with Saturating Actuators*. Springer, 2011.

[20] D. Liberzon, *Switching in Systems and Control*. Boston, MA, USA: Birkhaüser, 2003.

[21] H. Lin and P. J. Antsaklis, "Stability and stabilizability of switched linear systems: A survey of recent results," *IEEE Trans. Autom. Control*, vol. 54, no. 2, pp. 308–322, Feb. 2009.

[22] W. Xiang, H.-D. Tran, and T. T. Johnson, "Robust exponential stability and disturbance attenuation for discrete-time switched systems under arbitrary switching," *IEEE Trans. Autom. Control*, vol. 63, no. 5, pp. 1450–1456, May 2018.

[23] Z. Wu, M. Cui, P. Shi, and H. R. Karimi, "Stability of stochastic nonlinear systems with state-dependent switching," *IEEE Trans. Autom. Control*, vol. 58, no. 8, pp. 1904–1918, Aug. 2013.

[24] C. Briat, "Stability analysis and stabilization of stochastic linear impulsive, switched and sampled-data systems under dwell-time constraints," *Automatica*, vol. 74, pp. 279–287, Dec. 2016.

[25] H.-W. Jia and J. Zhao, "Output regulation of multiple heterogeneous switched linear systems," *Int. J. Automat. Comput.*, vol. 15, no. 4, pp. 492–499, 2018.

[26] L. Hou and G. Zong, "Stability of switched time-delay systems via mode-dependent average dwell time switching," *IEEE Access*, vol. 7, pp. 1174–1181, 2018.

[27] J. Yu, L. Wang, and G. Xie, "Disturbance rejection of switched systems subject to actuator saturation," *Trans. Inst. Meas. Control*, vol. 32, no. 6, pp. 603–634, 2010.

[28] W. Ni and D. Cheng, "Control of switched linear systems with input saturation," *Int. J. Syst. Sci.*, vol. 41, no. 9, pp. 1057–1065, 2010.
[29] Y. Ma, L. Fu, Y. Jing, and Q. Zhang, "Finite-time H_∞ control for a class of discrete-time switched singular time-delay systems subject to actuator saturation," *Appl. Math. Comput.*, vol. 261, pp. 264–283, Jun. 2015.
[30] X. Lin, S. Huang, S. Li, and Y. Zou, "Finite-time stabilisation of switched linear input-delay systems via saturating actuators," *IET Control Theory Appl.*, vol. 12, no. 15, pp. 2127–2137, Oct. 2018.
[31] Z. Zuo, Y. Li, Y. Wang, and H. Li, "Event-triggered control for switched systems in the presence of actuator saturation," *Int. J. Syst. Sci.*, vol. 49, no. 7, pp. 1478–1490, 2018.
[32] Q. Wang, L. Zhou, X. Ma, and C. Yang, "Disturbance rejection of singularly perturbed switched systems subject to actuator saturation," *Int. J. Robust Nonlinear Control*, vol. 28, no. 6, pp. 2231–2248, 2018.
[33] I. R. Petersen, "A stabilization algorithm for a class of uncertain linear systems," *Syst. Control Lett.*, vol. 8, no. 4, pp. 351–357, Mar. 1987.
[34] T. Wang, L. Xie, and C. E. de Souza, "Robust control of a class of uncertain nonlinear systems," *Syst. Control Lett.*, vol. 19, no. 2, pp. 139–149, 1992.
[35] X. Wei and L. Guo, "Composite disturbance-observer-based control and H_∞ control for complex continuous models," *Int. J. Robust Nonlinear Control*, vol. 20, no. 1, pp. 106–118, 2010.
[36] L. Guo and X.-Y. Wen, "Hierarchical anti-disturbance adaptive control for non-linear systems with composite disturbances and applications to missile systems," *Trans. Inst. Meas. Control*, vol. 33, no. 8, pp. 942–956, 2011.
[37] Y. Wei, J. Lu, and Z. Li, "Robust stabilization of uncertain switched nonlinear systems subject to actuator saturation," *IMA J. Math. Control Inf.*, vol. 31, no. 3, pp. 295–312, Sep. 2014.

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