On Input-to-State Stability of Switched Stochastic Nonlinear Systems under Extended Asynchronous Switching

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***Abstract*—An extended asynchronous switching model is investigated for a class of switched stochastic nonlinear retarded systems in the presence of both detection delay and false alarm, where the extended asynchronous switching is described by two independent and exponentially distributed stochastic processes, and further simplified as Markovian. Based on the Razumikhin-type theorem incorporated with average dwell time approach, the sufficient criteria for global asymptotic stability in probability and stochastic input-to-state stability are given, whose importance and effectiveness are finally verified by numerical examples.**

***Index Terms*—SSNLRS, extended asynchronous switching, Markovian theory, Razumikhin-type theorem, input-to-state stability.**

I. Introduction

SWITCHED systems has been a topic of increasing interest in recent years [1]–[3]. Informally, a switched system is a family of dynamical subsystems with a switching rule governing the switching between them. Such models are used to describe many physical and man-made systems with jumping parameters or changing environmental factors [4]–[6]. During the last decades, the control of switched system has received considerable attentions [7]–[14].

Recently, the stability problem at the boundary of switched systems and time delay systems has received much attention [15]–[22]. To design the mode-dependent controller for a practical switched system, one should get the switching signal of the plant. However, due to unknown abrupt phenomena such as component and interconnection failures, delays in the detection of the switching signal have to be considered, thus enabling the so-called conventional asynchronous switching [19]. Roughly speaking, the "conventional asynchronous switching" is caused by the detection delay of switching signal which results in the mismatched period of designed controller in each subsystem and then deteriorates the system performance and then will deteriorate the system performance. This marks the importance of the stability analysis and control synthesis under asynchronous switching. Many efforts have been made, for example, state feedback stabilization, input-to-state stabilization, output feedback stabilization, fault-tolerant control and asynchronous filtering, etc. [15]–[18], [23]–[26], just to name a few. However, only the detection delay is considered in these works. In reality, the existence of environmental noises, disturbances, and small modelling uncertainties means that false alarm (or detection error) is inevitable, which fails existing results for asynchronous switching with only detection delays. In what follows the control for a class of new asynchronous switching system with simultaneous considering the detection delays and the false alarms will be considered. To distinguish it from the conventional asynchronous switching, it is called the extended asynchronous switching system. Compared to the conventional asynchronous switching, the developed extended asynchronous switching in this paper can better reflect the actual situation in practical switched system control. In addition, it has shown in [27] that the non-zero detection delay can make a closed-loop system unstable. Therefore, the existence of false alarm will inevitably further decrease the control performance. Thus, the so-called extended asynchronous switching justifies its importance. However, the coupled relationship between the true switching signal and the random detection as well as the the false alarm also increase the complexity and difficulty of stability analysis for such system. Moreover, to date switched stochastic nonlinear retarded systems (SSNLRS) under extended asynchronous switching have received little attention. All those motivate our present study.

To overcome the challenges faced and the limitations found in the literature, this paper investigates the stochastic input-to-state stability for a class of SSNLRS under extended asynchronous switching. Firstly, an extended asynchronous switching model is developed to describe the more realistic situation of switching signal detection in the presence of both the random detection delay and the false alarm. Given that

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the existence of various noises and uncertainties in practical control, the detection delay and false alarm in practice are essentially uncertain and random. To describe such phenomena, two stochastic processes are needed [28], [29]. As described in [28], we define the detected switching signal conditionally on the true one, and use two independent exponentially distributed stochastic processes to describe the detection delay and the false alarm, respectively. Then a practical model in which the Markovian theory is used to describe the probability characteristic of the extended asynchronous switching phenomenon. And the Razumikhin-type stability criteria based on average dwell time approach are developed for the proposed extended asynchronous switching system. Finally, the importance and effectiveness are validated by the simulation studies.

The remainder of the paper is organized as follows. The extended asynchronous switching model is formulated and necessary definitions are given in Section II. The main results are then discussed in Section III, with an example given in Section IV. Section V concludes the paper. Some technical details are included in Appendices.

Notations. Throughout this paper, $\mathbb{R}_+$ and $\mathbb{N}_+$ denote the set of nonnegative real numbers and positive integer, respectively. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, n-dimensional real space and $n \times m$-dimensional real matrix space. For vector $x \in \mathbb{R}^n$, $|x|$ denotes the Euclidean norm $|x| = (\sum_{i=1}^n x_i^2)^{\frac{1}{2}}$. Let $\tau \geq 0$ and $C([-\tau, 0]; \mathbb{R}^n)$ denote the family of all continuous $\mathbb{R}^n$-valued functions φ on $[-\tau, 0]$ with the norm $\|\varphi\| = \sup\{|\varphi(\theta)| : -\tau \leq \theta \leq 0\}$. Let $C^b_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ be the family of all $\mathcal{F}_0$-measurable bounded $C([-\tau, 0]; \mathbb{R}^n)$-valued random variables $\xi = \{\xi(\theta) : -\tau \leq \theta \leq 0\}$. For $p > 0$ and $t \geq 0$, let $L^p_{\mathcal{F}_t}([-\tau, 0]; \mathbb{R}^n)$ denote the family of all $\mathcal{F}_t$-measurable $C([-\tau, 0]; \mathbb{R}^n)$-valued random variables $\phi = \{\phi(\theta) : -\tau \leq \theta \leq 0\}$ such that $\sup_{-\tau\leq\theta\leq 0} \mathbb{E}\{ |\phi(\theta)|^p\} < \infty$. The transpose of vectors and matrices and the complement of set are denoted by a superscript T and C, respectively. $C^{i,k}$ denotes all the functions with ith continuously differentiable first component and kth continuously differentiable second component.

II. Problem Formulation and Preliminaries

Consider the following switched stochastic nonlinear retarded systems (SSNLRS)

$$\mathrm{d}x = f(t, x_t, \nu, \sigma)\mathrm{d}t + g(t, x_t, \nu, \sigma)\mathrm{d}w \tag{1}$$

with the initial state $x_0 = \{x(\theta) : -\tau \leq \theta \leq 0\} = \xi \in C^b_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$, where $x(t) \in \mathbb{R}^n$ is the state vector, $x_t = \{x(t + \theta) : -\tau \leq \theta \leq 0\}$ is $C([-\tau, 0]; \mathbb{R}^n)$-valued random process, $\nu(t) \in \mathcal{L}^l_\infty$ is the control input. $\mathcal{L}^l_\infty$ denotes the set of all the measurable and locally essentially bounded input $\nu(t) \in \mathbb{R}^l$ on $[t_0, \infty)$ with the norm

$$\begin{cases} \|\nu(s)\| = \inf\limits_{\mathcal{A}\subset\Omega, \mathbb{P}(\mathcal{A})=0} \sup\{|\nu(\omega, s)| : \omega \in \Omega\backslash\mathcal{A}\} \\ \|\nu(s)\|_{[t_0,\infty)} = \sup\limits_{s\in[t_0,\infty)} \|\nu(s)\|. \end{cases} \tag{2}$$

$w(t)$ is the m-dimensional Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq t_0}, \mathbb{P})$, with Ω being a sample space, $\mathcal{F}$ being a σ-field, $\{\mathcal{F}_t\}_{t\geq t_0}$ being a filtration and $\mathbb{P}$ being a probability measure. $\sigma : [t_0, \infty) \to \mathcal{S}$ ($\mathcal{S}$ is the index set, and may be infinite) is the switching law and is right hand continuous and piecewise constant on t. $\sigma(t)$ discussed in this paper is time dependent, and the corresponding switching times sequence is $\{t_l\}_{l\geq 0}$. The i_lth subsystems will be activated at time interval $[t_l, t_{l+1})$. Specially, when $t = t_0$ (t_0 is the initial time), suppose $\sigma_0 = \sigma(t_0) = i_0 \in \mathcal{S}$. Moreover, $f : \mathbb{R}_+ \times C([-\tau, 0]; \mathbb{R}^n) \times \mathbb{R}^l \times \mathcal{S} \to \mathbb{R}^n$ and $g : \mathbb{R}_+ \times C([-\tau, 0]; \mathbb{R}^n) \times \mathbb{R}^l \times \mathcal{S} \to \mathbb{R}^{n\times m}$ are continuous with respect to t, $x(t)$, $u(t)$, and satisfy uniformly locally Lipschitz condition with respect to $x(t)$, $u(t)$, and for any $i \in \mathcal{S}$, $f(t, 0, 0, i) \equiv 0$, $g(t, 0, 0, i) \equiv 0$.

Given that the true switching signal is not available for the controller design in practical, in what follows, we are concerned with the stability analysis of systems (1) under the following state feedback control law,

$$\nu = h(t, x_t, u, \sigma'). \tag{3}$$

where $\sigma' = \sigma'(t)$ is the detected switching signal, $u(t) \in \mathcal{L}^k_\infty$ is the reference input, and $h : \mathbb{R}_+ \times C([-\tau, 0]; \mathbb{R}^n) \times \mathbb{R}^k \times \mathcal{S} \to \mathbb{R}^l$ is measurable function with $h(t, 0, 0, i) \equiv 0$, for any $i \in \mathcal{S}$. For convenience, denote

$$\bar{f}(t, x_t, u, \sigma, \sigma') = f(t, x_t, h(t, x_t, u, \sigma'), \sigma)$$
$$\bar{g}(t, x_t, u, \sigma, \sigma') = g(t, x_t, h(t, x_t, u, \sigma'), \sigma)$$

and further, let $\bar{f}_{ij}(t, x_t, u(t))$ and $\bar{g}_{ij}(t, x_t, u(t))$ denote the corresponding function $\bar{f}(t, x_t, u(t), i, j)$ and $\bar{g}(t, x_t, u(t), i, j)$, respectively. Moreover, we assume that $\bar{f}$, $\bar{g}$ satisfy the local Lipschitz condition and the linear growth condition, hence for the closed-loop system

$$\mathrm{d}x = \bar{f}(t, x_t, u, \sigma, \sigma')\mathrm{d}t + \bar{g}(t, x_t, u, \sigma, \sigma')\mathrm{d}w \tag{4}$$

and there exists an unique solution on $t \geq -\tau$.

In practical control systems, due to the existence of environmental noises, disturbances, and small modelling uncertainties, non-zero detection delays and false alarms may occur randomly in the mode detection of the plant. In this case, as in reference [28], the following model is assumed to describe the characteristic of $\sigma'(t)$.

Assumption 2.1: [28] The values of $\sigma(t)$ and $\sigma'(t)$ can be divided into two cases: the quiescent case $\sigma(t) = \sigma'(t) = i$ and the transient case $\sigma(t) = i$, $\sigma'(t) = j$, $j \neq i$. In the first case, only the true modes switches and false alarms may occur. The later case corresponds to the detection delay or to the recovery from a false alarm. The only possible switch is thus a switch of $\sigma'(t)$ from j to i, corresponding to the end of the transient, and this switch occurs on the average after $\frac{1}{\pi^0_{ji}}$ seconds. In mathematic,

Case I. When $\sigma(t)$ has switched from i to j, $\sigma'(t)$ follows with a delay d that is an independent exponentially distributed random variable with mean $\frac{1}{\pi^0_{ij}}$. This is written as

$$\mathbb{P}\{\sigma'(t + \Delta) = j|\sigma'(s) = i, s \in [t^*, t], \sigma(t^*) = j, \sigma(t^{*-}) = i\} = \begin{cases} \pi^0_{ij}\Delta + o(\Delta), \ i \neq j \\ 1 + \pi^0_{ii}\Delta + o(\Delta), \ i = j. \end{cases} \tag{5}$$

The entries of the matrix, $\Pi^0 = [\pi^0_{ij}]_{N\times N} \in \mathbb{R}^{N\times N}$, are evaluated from observed sample paths, and

$$\pi^0_{ii} = -\sum_{j\neq i} \pi^0_{ij}, (\pi^0_{ij} \geq 0, i \neq j). \tag{6}$$

Case II. When $\sigma(t)$ remains at i, $\sigma'(t)$ has transitioned from i to j occasionally. An independent exponential distribution with rate π^1_{ij} is again assumed

$$\begin{aligned} &\mathbb{P}\{\sigma'(t+\Delta) = j|\sigma'(s) = i, s \in [t^*, t]\} \\ &= \begin{cases} \pi^1_{ij}\Delta + o(\Delta), \ i \neq j \\ 1 + \pi^1_{ii}\Delta + o(\Delta), \ i = j \end{cases} \end{aligned} \tag{7}$$

with a matrix, $\Pi^1 = [\pi^1_{ij}]_{N\times N} \in \mathbb{R}^{N\times N}$, of false alarm rates, which can also be valued from observed sample paths, and

$$\pi^1_{ii} = -\sum_{j\neq i} \pi^1_{ij}, (\pi^1_{ij} \geq 0, i \neq j). \tag{8}$$

According to [28], it then follows from Assumption 2.1 that:

Property 2.1: According to Assumption 2.1, the greater π^0_{ij} is the faster detection response speed is, and the smaller π^1_{ij} is the less of the number of false alarms is, where $i, j \in \mathcal{S}$. When $\pi^0_{ij} \to \infty$ and $\pi^1_{ij} = 0$, the detection for the actual switching signal is perfect.

From Assumption 2.1, under any time interval $[t_m, t_{m+1})$, where $t_m, t_{m+1} \in \{t_l\}_{l\geq 0}$, the number of switches of $\sigma'(t)$ can only be the following two cases: $2k+1$ and $2k$, where $k \geq 0$ is the switch number of $\sigma'(t)$ which caused by false alarm. We assume $\sigma(t_m) = i_m$. In the first case, $\sigma'(t)$ will first switch to i_m with responding to transient case, i.e., detection delay process, and the detection delay doesn't equal to zero. After $\sigma'(t) = i_m, t \in (t_m, t_{m+1})$, if a false alarm occurs, the next switch is that $\sigma'(t)$ switch to i_m (recovery from the false alarm mode). Thus, before time t_{m+1}, the total switch number of $\sigma'(t)$ is $2k+1$. On the other hand, if the detection delay is zero, i.e, $\sigma'(t_m) = \sigma(t_m) = i_m$, then total switch number of $\sigma'(t)$ on $[t_m, t_{m+1})$ will be $2k$.

Let $\{t'_l\}_{l\geq 0}$ denote the switching times sequence of $\sigma'(t)$, with $t_0 = t'_0$ and $\sigma'(t'_0) = \sigma(t_0)$. For any $i \in \mathbb{N}_+ \cup \{0\}$, let $N(t_{i+1}, t_i)$ denote the number of switches of $\sigma'(t)$ on $[t_i, t_{i+1})$. Moreover, as shown in Fig.1, we subdivide the sequence $\{t'_l\}_{l\geq 0}$ into a sequence of subsets, i.e., $\{t'_l\}_{l\geq 0} = \bigcup_i \{t'_{i1}, t'_{i2}, \cdots, t'_{iN(t_{i+1},t_i)}\}$, such that $\{t'_{i1}, t'_{i2}, \cdots, t'_{iN(t_{i+1},t_i)}\} \subset [t_i, t_{i+1})$. In the sequel, we assume that $\sigma(t_i^-) = \sigma'(t_i^-)$ and $\sigma'(t) = \sigma(t) = \sigma(t_0) = i_0$, for any $t \in (t_0, t_1)$. Note that, for any $i \in \mathbb{N}_+ \cup \{0\}$, $\sigma(t_i^-) = \sigma'(t_i^-)$ means that the switches between the subsystems of switched system occur in the case that the controller and the

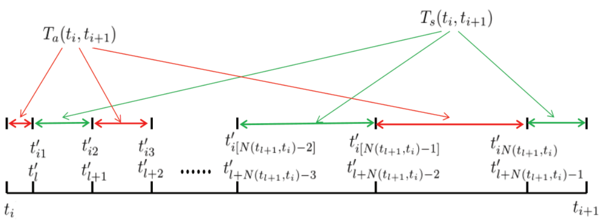

Fig. 1: The re-definition of $\{t'_l\}_{l\geq 0}$ on interval $[t_i, t_{i+1})$.

system operate synchronously. The hypothesis is commonly employed in the context in asynchronous switching systems, in which there always exists the period that the controller and the system run synchronously [15]–[19].

For any $i \in \mathbb{N}$, if the detection delay is non-zero, then the controller mode is strictly synchronous with the system on the following time intervals: $[t'_{i1}, t'_{i2})$, $[t'_{i3}, t'_{i4})$, $\cdots$, $[t'_{iN(t_{i+1},t_i)}, t_{i+1})$. For simplicity, we define $T_s(t_0, t_1) = [t_0, t_1)$, $T_s(t_i, t_{i+1}) = \bigcup_{j=1,3,\cdots,N(t_{i+1},t_i)} [t'_{ij}, t'_{i(j+1)})$, and $T_a(t_i, t_{i+1}) = \bigcup_{j=0,2,\cdots,N(t_{i+1},t_i)-1} [t'_{ij}, \ t'_{i(j+1)})$, where $t'_{i(N(t_{i+1},t_i)+1)} = t_{i+1}$, $t'_{i0} = t_i$. However, if the detection delay is equal to zero, then the controller mode is strictly synchronous with the system on the following time intervals: $[t_i, t'_{i1})$, $[t'_{i2}, t'_{i3})$, $\cdots$, $[t'_{iN(t_{i+1},t_i)}, t_{i+1})$. In this case, $T_s(t_i, t_{i+1}) = \bigcup_{j=0,2,\cdots,N(t_{i+1},t_i)} [t'_{ij}, t'_{i(j+1)})$, and $T_a(t_i, t_{i+1}) = \bigcup_{j=1,3,\cdots,N(t_{i+1},t_i)-1} [t'_{ij}, t'_{i(j+1)})$. Clearly, it has $T_s(t_i, t_{i+1}) \bigcap T_a(t_i, t_{i+1}) = \varnothing$, $[t_i, t_{i+1}) = T_a(t_i, t_{i+1}) \bigcup T_s(t_i, t_{i+1})$. In the sequel, we let $T_a(t-s)$ denote the length of $T_a(t, s)$, for any $t \geq s \geq t_0$.

To simplify the expression, the next definitions are needed.

Definition 2.1: [30] A function $\alpha : \mathbb{R}_+ \to \mathbb{R}_+$ is said to belong to class $\mathcal{K}$ if α is continuous, strictly increasing and $\alpha(0) = 0$. If α is also unbounded, then it is of class $\mathcal{K}_\infty$. Class $\mathcal{CK}_\infty$ and $\mathcal{VK}_\infty$ function are the two subsets of class $\mathcal{K}_\infty$ functions that are concave and convex, respectively. A function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is of class $\mathcal{KL}$, if $\beta(\cdot, t)$ is of class $\mathcal{K}$ in the first argument for each fixed $t \in \mathbb{R}_+$ and $\beta(s, t)$ decreases to 0 as $t \to +\infty$ for each fixed $s \geq 0$.

Definition 2.2: [1] For any given constants $\tau^* > 0$ and N_0, let $N_\sigma(t, s)$ denote the switch number of $\sigma(t)$ in $[s, t)$, for any $t > s \geq t_0$, and let

$$S[\tau^*, N_0] = \{\sigma(\cdot) : \ N_\sigma(t, s) \leq N_0 + \frac{t-s}{\tau^*}, \ \forall s \in [t_0, t)\}.$$

Then τ^* is called the average dwell-time of $S[\tau^*, N_0]$, and $\tau_\sigma \triangleq \sup_{t\geq t_0} \sup_{t>s\geq t_0} \frac{t-s}{N_\sigma(t,s)-N_0}$ is called the average dwell-time of $\sigma(\cdot)$.

Definition 2.3: [32] The solution of system (4) is said to be stochastic input-to-state stable (SISS), if for any $\varepsilon > 0$, there exist $\mathcal{KL}$ function $\beta(\cdot, \cdot)$ and $\mathcal{K}$ function $\gamma(\cdot)$, such that

$$\mathbb{P}\{|x(t)| < \beta(\mathbb{E}\{\|\xi\|\}, t - t_0) + \gamma(\|u\|_{[t_0,t)})\} \geq 1 - \varepsilon \tag{9}$$

for any $t \geq t_0$ and $\xi \in C^b_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$, where $\|u\|_{[t_0,t)} = \sup_{s\in[t_0,t)} \|u(s)\|$. Specifically, if $u \equiv 0$, under the hypothesis (9), the system is globally asymptotically stable in probability (GASiP).

Definition 2.4: [31] The solution of system (4) is said to be pth moment ISS, if for all $\xi \in C^b_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ and $i_0 \in \mathcal{S}$, there exist $\mathcal{KL}$ function $\beta(\cdot, \cdot)$ and $\mathcal{K}$ function $\gamma(\cdot)$, such that

$$\mathbb{E}\{|x(t)|^p\} \leq \beta(\mathbb{E}\{\|\xi\|^p\}, t - t_0) + \gamma(\|u\|_{[t_0,t)}), \ \forall t \geq t_0. \tag{10}$$

III. Stability Analysis under Extended Asynchronous Switching

Based on the above developed extended asynchronous switching model, this section presents the stability criteria for the SSNLRS under extended asynchronous switching controller. By using Razumikhin-type theorem and average dwell time approach, we give the sufficient conditions for internal stability, i.e., globally asymptotically stability in probability and pth moment exponentially stability. Using the internal stability criteria, then the external stability criteria are developed, including SISS and pth moment ISS.

For the sake of simplify expression, denote $\bar{\pi}^0 \triangleq \max_{i\in\mathcal{S}}\{|\pi_{ii}^0|\}$, $\tilde{\pi}^0 \triangleq \max_{i,j\in\mathcal{S}}\{\pi_{ij}^0\}$, $\bar{\pi}^1 \triangleq \max_{i\in\mathcal{S}}\{|\pi_{ii}^1|\}$, $\tilde{\pi}^1 \triangleq \max_{i,j\in\mathcal{S}}\{\pi_{ij}^1\}$, $\underline{\pi}^0 \triangleq \min_{i\in\mathcal{S}}\{|\pi_{ii}^0|\}$, $\underline{\pi}^1 \triangleq \min_{i\in\mathcal{S}}\{|\pi_{ii}^1|\}$.

Theorem 3.1: Let $\varsigma = \sup_{l\in\mathbb{N}_+}\{t_l - t_{l-1}\} < \infty$. If there exist functions $\alpha_1 \in \mathcal{K}_\infty$, $\alpha_2 \in \mathcal{CK}_\infty$, $\mu \geq 1$, $q > 1$, $\lambda_1 > 0$, $\lambda_2 \geq 0$, and $V(x(t), t, \sigma(t), \sigma'(t)) \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \times \mathcal{S} \times \mathcal{S}; \mathbb{R}_+)$, such that

(i). for all $t \geq t_0 - \tau$,

$$\alpha_1(|x(t)|) \leq V(x(t), t, \sigma(t), \sigma'(t)) \leq \alpha_2(|x(t)|). \tag{11}$$

(ii). there exists $\bar{\lambda}_1 \in (0, \lambda_1)$ such that

$$\begin{aligned}&\mathbb{E}\{\mathcal{L}V(\varphi(\theta), t, \sigma(t), \sigma'(t))\}\\ &\leq \begin{cases} -\lambda_1\mathbb{E}\{V(\varphi(0), t, \sigma(t), \sigma'(t))\}, \\ \qquad\qquad \text{if } t \in T_s(t_l, t_{l+1}),\ l \in \mathbb{N}_+ \cup \{0\} \\ \lambda_2\mathbb{E}\{V(\varphi(0), t, \sigma(t), \sigma'(t))\}, \\ \qquad\qquad \text{if } t \in T_a(t_l, t_{l+1}),\ l \in \mathbb{N}_+ \end{cases}\end{aligned} \tag{12}$$

provided those $\varphi \in L^p_{\mathcal{F}_t}([-\tau, 0]; \mathbb{R}^n)$ satisfying that

$$\min_{i,j\in\mathcal{S}} \mathbb{E}\{V(\varphi(\theta), t+\theta, i, j)\} \leq q\mathbb{E}\{V(\varphi(0), t, \sigma(t), \sigma'(t))\} \tag{13}$$

where

$$e^{\bar{\lambda}_1\tau} \leq q. \tag{14}$$

(iii). and for any $i, j \geq 1$, the candidate function $V(x(t), t, \sigma(t), \sigma'(t))$ satisfies

$$\begin{cases} \mathbb{E}\{V(x(t'_{ij}), t'_{ij}, \sigma(t_i), \sigma'(t'_{ij}))\} \\ \qquad \leq \mu\mathbb{E}\{V(x(t'_{ij}), t'_{ij}, \sigma(t_i), \sigma'(t'_{i(j-1)}))\} \\ \mathbb{E}\{V(x(t_i), t_i, \sigma(t_i), \sigma'(t_i))\} \\ \qquad \leq \mu\mathbb{E}\{V(x(t_i), t_i, \sigma(t_{i-1}), \sigma'(t'_{(i-1)N(t_i,t_{i-1})}))\} \end{cases} \tag{15}$$

(iv). it exists $\bar{\lambda}_2 \in (\lambda_2, \infty)$ such that

$$\bar{\lambda}_1 + \bar{\lambda}_2 - \underline{\pi}^0 < 0 \tag{16}$$

for any $i \geq 1$ and $j = 1, 2, \cdots, N(t_{i+1}, t_i)$, with $t'_{i0} = t_i$, $t'_{0N(t_1,t_0)} = t'_{00} = t_0$, further, the average dwell time τ^* satisfies $\tau^* > \frac{\ln(\mu M)}{\bar{\lambda}_1}$, where

$$\begin{aligned} M = (1+\mu)[\frac{-\bar{\pi}^0}{\bar{\lambda}_1 + \bar{\lambda}_2 - \underline{\pi}^0} + (\frac{-\bar{\pi}^0}{\bar{\lambda}_1 + \bar{\lambda}_2 - \underline{\pi}^0})^{\frac{1}{2}}] \\ \times e^{[(\mu^2 - \frac{\bar{\pi}^0}{\bar{\lambda}_1 + \bar{\lambda}_2 - \underline{\pi}^0})(N-1)-2]\tilde{\pi}^1\varsigma} \end{aligned} \tag{17}$$

then system (4) with $u \equiv 0$ is GASiP.

Proof: Please see Appendix B. ∎

Remark 3.1: In Theorem 3.1, the assumptions (11), (13), (14) and (15) are common conditions in the stability analysis of switched stochastic time-delay systems [35]. The condition (12) is also commonly employed in the asynchronous switched deterministic systems [18], while (16) is set to restrict the conditions that the system (4) need to be satisfied under the existence of detection delay and false alarm.

Remark 3.2: For the detection of $\sigma(t)$, consider the following two special cases. First, if Π^0 and Π^1 are set to ∞ and zero, respectively, there is no detection delay and no false alarm in the mode detection, the closed-loop systems is a synchronous case. In this case, the conditions (12) and (16) hold almost surely. On the other hand, if Π^1 is set to zero while $\Pi^0 < \infty$, the situation corresponds to only a detection delay, and $\underline{\pi}^0 < \infty$, $\tilde{\pi}^1 = 0$. Hypothesis (16) restricts the necessary condition that the closed-loop systems need to be satisfied under this case.

Remark 3.3: According to (16), (17) and the average dwell time τ^* in Theorem 3.1, one can see that the stability of the extended asynchronous switching systems can be guaranteed by a sufficient small mismatched time interval and a sufficient large average dwell time. Given that the mismatched time interval in the developed extended asynchronous switching framework is usually caused by: the size of detection delay, the frequency of occurrence from false alarms, and the length of the recovery time from a false mode, it is further explained as follows:

Case I. For any fixed $\bar{\lambda}_1$, μ, ς and $\tilde{\pi}^1$, a larger instability margin λ_2 (or $\bar{\lambda}_2$) will be compensated by a larger $\underline{\pi}^0$ and/or a larger average dwell time τ^*. Since $\underline{\pi}^0 = \min_{i\in\mathcal{S}}\{|\pi_{ii}^0|\}$, a larger $\underline{\pi}^0$ can be obtained by increasing $|\pi_{ii}^0|$ or decreasing π_{ii}^0. The larger $|\pi_{ii}^0|$ is the smaller of the detection delay for mode i is. Thus, when π_{ii}^0 increases, if $\bar{\pi}^0 = \max_{i,s\in\mathcal{S}}\{\pi_{ij}^0\}$ is non-increase, a lager instability margin can be compensated by a small detection delay; however if $\bar{\pi}^0$ is also increased, a larger average dwell time τ^* will work, and the larger instability margin will be compensated by a smaller detection delay and a larger average dwell time τ^*.

Case II. When $\bar{\lambda}_1$, μ and ς are fixed, and we assume $\underline{\pi}^0$ and $\bar{\pi}^0$ do not change through a fixed constant π_{ij}^0 $(i, j \in \mathcal{S})$, then if the instability margin λ_2 (or $\bar{\lambda}_2$) increases, M will also increase. In this case, the larger M can be compensated by a smaller $\tilde{\pi}^1$ or a larger average dwell time τ^*. Note that, a fixed constant π_{ij}^0 $(i, j \in \mathcal{S})$ means that the time costs of the detection of true modes and the recovery from a false mode do not change on the average. Given that $\tilde{\pi}^1 = \max_{i,j\in\mathcal{S}}\{\pi_{ij}^1\}$, $\tilde{\pi}^1$ can be reduced by decreasing π_{ij}^1. Then the number of false alarms will decrease, and consequently, the mismatched time from false alarms will also decrease, which can well compensate the impact of larger instability margin.

Using the GASiP criterion in Theorem 3.1, one can further obtain the following SISS conditions.

Theorem 3.2: Let $\varsigma = \sup_{l\in\mathbb{N}_+}\{t_l - t_{l-1}\} < \infty$. If there exist functions $\gamma \in \mathcal{K}$, $\alpha_1 \in \mathcal{VK}_\infty$, $\alpha_2 \in \mathcal{CK}_\infty$, $\mu \geq 1$, $q > 1$, $\lambda_1 > 0$, $\lambda_2 \geq 0$, $\bar{\lambda}_1 \in (0, \lambda_1)$, $\bar{\lambda}_2 \in (\lambda_2, \infty)$, and $V(x(t), t, \sigma(t), \sigma'(t)) \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \times \mathcal{S} \times \mathcal{S}; \mathbb{R}_+)$, such that hypotheses i), iii), iv) in Theorem 3.1 hold, and

$$|\varphi(0)| \geq \gamma(\|u\|_{[t_0,\infty)}) \Rightarrow \mathbb{E}\{\mathcal{L}V(\varphi(\theta), t, \sigma(t), \sigma'(t))\} \leq \begin{cases} -\lambda_1 \mathbb{E}\{V(\varphi(0), t, \sigma(t), \sigma'(t))\}, \\ \qquad t \in T_s(t_l, t_{l+1}),\ l \in \mathbb{N}_+ \cup \{0\} \\ \lambda_2 \mathbb{E}\{V(\varphi(0), t, \sigma(t), \sigma'(t))\}, \\ \qquad t \in T_a(t_l, t_{l+1}),\ l \in \mathbb{N}_+ \end{cases} \tag{18}$$

provided those $\varphi \in L^p_{\mathcal{F}_t}([-\tau, 0]; \mathbb{R}^n)$ satisfying that

$$\min_{i,j\in\mathcal{S}} \mathbb{E}\{V(\varphi(\theta), t+\theta, i, j)\} \leq q\mathbb{E}\{V(\varphi(0), t, \sigma(t), \sigma'(t))\} \tag{19}$$

where

$$e^{\bar{\lambda}_1 \tau} \leq q. \tag{20}$$

Then, system (4) is SISS.

Proof: The proof is similar to Theorem 4.1 of [27] and is thus omitted. ■

Remark 3.4: Despite the similarities of Theorem 3.2 in this present paper and Theorem 4.1 in our early work [27], the following essential differences are observed.

(1). Due to the existence of mismatched time interval which caused by detection delays and false alarms, after the state trajectory enters the set $\mathfrak{B}$, there still exists a chance to leave it. This complicates the system and is different from the normal asynchronous case in [27].

(2). The system in this paper is deterministic switched system while the system in [27] is Markovian switching.

(3). [27] considers only the detection delay while this work consider both the non-zero detection delay and the false alarm. The inclusion of false alarm makes the extended asynchronous switching model in this paper more practical.

Corollary 3.1: Under the assumptions in Theorem 3.1 (Theorem 3.2), if functions α_1, α_2 satisfy $\alpha_1(s) = c_1 s^p$, $\alpha_2(s) = c_2 s^p$, where c_1 and c_2 are positive numbers, then system (4) is pth moment exponentially stable with $u \equiv 0$ (pth moment ISS), for all $\tau^* > \frac{\ln(\mu M)}{\bar{\lambda}_1}$.

IV. Simulation Study

In this section the general Razumikhin-type theorems established in the previous section will be extended to deal with the input-to-state stability of switched stochastic nonlinear delay system (SSNLDS).

For a simulation purpose, consider a special class of switched stochastic perturbed system

$$\mathrm{d}x = [A_\sigma x + B_\sigma \nu]\mathrm{d}t + g(t, x(t-d(t)), \sigma)\mathrm{d}w \tag{21}$$

where $g : \mathbb{R}_+ \times \mathbb{R}^n \times \mathcal{S} \to \mathbb{R}^n$ is unknown nonlinear function satisfying the local Lipschitz condition and the linear growth condition, and $\|g(t, x(t-d(t)), i)\|_2 \leq \|U_i x(t-d(t))\|_2$, $\|\cdot\|_2$ denotes the 2-norm, for any $i \in \mathcal{S}$. U_i is known real constant matrix, and $0 \leq d(t) \leq \tau$. Design $\nu(t) = K_{\sigma'(t)} x(t) + u(t)$. Then, the closed-loop system is

$$\mathrm{d}x = [A_\sigma x + B_\sigma K_{\sigma'} x + B_\sigma u]\mathrm{d}t + g(t, x(t-d(t)), \sigma)\mathrm{d}w \tag{22}$$

From Corollary 3.1, one has the following corollary.

Corollary 4.1: System (22) is 2th moment ISS for all $\tau^* > \frac{\ln(\mu M)}{\hat{\lambda}_1}$, where

$$M = (1+\mu)\left[\frac{-\bar{\pi}^0}{\hat{\lambda}_1 + \hat{\lambda}_2 - \underline{\pi}^0} + \left(\frac{-\bar{\pi}^0}{\hat{\lambda}_1 + \hat{\lambda}_2 - \underline{\pi}^0}\right)^{\frac{1}{2}}\right] \times e^{[(\mu^2 - \frac{\bar{\pi}^0}{\hat{\lambda}_1 + \hat{\lambda}_2 - \underline{\pi}^0})(N-1)-2]\bar{\pi}^1 \varsigma} \tag{23}$$

if for all $i, j \in \mathcal{S}$, there exist $X_{ij} = X_{ij}^T > 0$, $\lambda_1 > 0$, $\lambda_2 \geq 0$, $\lambda_{10} \geq 0$, $\lambda_{20} \geq 0$ such that (24)-(27) hold, i.e.,

$$\begin{bmatrix} -\lambda_{10} X_{ii} & X_{ii} U_i^T \\ * & -\frac{2}{\beta_1} I \end{bmatrix} \leq 0 \tag{25}$$

$$\begin{bmatrix} \Pi_2 & X_{ij} & X_{ij} \\ * & -\frac{1}{\pi_{ji}^0} X_{ii} & 0 \\ * & * & -Q^{-1} \end{bmatrix} \leq 0 \tag{26}$$

$$\begin{bmatrix} -\lambda_{20} X_{ij} & X_{ij} U_i^T \\ * & -\frac{2}{\beta_2} I \end{bmatrix} \leq 0 \tag{27}$$

where $\Pi_1 = A_i X_{ii} + X_{ii} A_i^T + Y_{ii}^T B_i^T + B_i Y_{ii} + \varepsilon_1 B_i B_i^T + (\lambda_1 + \pi_{ii}^1) X_{ii}$, $\Pi_2 = X_{ij} A_i^T + A_i X_{ij} + Y_{ij}^T B_i^T + B_i Y_{ij} + \varepsilon_2 B_i B_i^T - (\lambda_2 + \pi_{ji}^0) X_{ij}$, $\varepsilon_1, \varepsilon_2 > 0$; there exist $q > 1$, such that $\bar{\lambda}_1 = \lambda_1 - q\lambda_{10} > 0$; and

$$e^{\hat{\lambda}_1 \tau} < q \tag{28}$$

$$\hat{\lambda}_1 + \hat{\lambda}_2 - \underline{\pi}^0 < 0 \tag{29}$$

where $\bar{\lambda}_2 = \lambda_2 + q\lambda_{20}$, $\hat{\lambda}_1 \in (0, \bar{\lambda}_1)$ and $\hat{\lambda}_2 \in (\bar{\lambda}_2, \infty)$.

Proof: See Appendix C. ■

Example 4.1: Take the following parameters for system (22):

$$A_1 = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix},\ B_1 = \begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix},\ U_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 3 & 0 \\ 2 & 3 \end{bmatrix},\ B_2 = \begin{bmatrix} -2 & 1 \\ 0 & 2 \end{bmatrix},\ U_2 = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}$$

and $g(t, \bar{x}(t), 1) = [0.1\cos(t) x_1(t-d(t)), 0]^T$, $g(t, \bar{x}(t), 2) = [0.1\sin(t) x_2(t-d(t)), 0.1 x_1(t-d(t))]^T$, where $\bar{x}(t) = x(t-d(t)) d(t) = 0.2\sin(t)$ with $\tau = 0.2$, and

$$\Pi^0 = \begin{bmatrix} -100 & 100 \\ 80 & -80 \end{bmatrix}, \Pi^1 = \begin{bmatrix} -0.2 & 0.2 \\ 0.2 & -0.2 \end{bmatrix}.$$

Then conditions (24), (25), (26) and (27) can be satisfied with $\lambda_1 = 10$, $\lambda_2 = 5.9182$, $\lambda_{10} = 0.1$, $\lambda_{20} = 0.1$, $\varepsilon_1 = 7$, $\varepsilon_2 = 2$, $\mu = 1.38$, $\beta_1 = 1.6956$, $\beta_2 = 1.5955$, and, moreover,

$$P_{11} = \begin{bmatrix} 0.4491 & -0.0001 \\ -0.0001 & 0.4484 \end{bmatrix}$$

$$P_{12} = \begin{bmatrix} 0.5877 & -0.0005 \\ -0.0005 & 0.5877 \end{bmatrix}$$

$$P_{21} = \begin{bmatrix} 0.5877 & -0.0005 \\ -0.0005 & 0.5880 \end{bmatrix}$$

$$P_{22} = \begin{bmatrix} 0.4483 & -0.0034 \\ -0.0034 & 0.4509 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} 28.9391 & 7.3873 \\ 7.4055 & 9.2980 \end{bmatrix}, K_2 = \begin{bmatrix} 5.8994 & -3.4292 \\ -3.3107 & -6.9532 \end{bmatrix}$$

and $Q = diag[0.1873, 0.1873]$.

Take $q = 7.5$, then $\bar{\lambda}_1 = 9.25$ and $\bar{\lambda}_2 = 6.6682$, and further, conditions (28) and (29) can be satisfied with $\hat{\lambda}_1 = 9.1575$,

$$\begin{bmatrix} \Pi_1 & X_{ii} & X_{ii} & \cdots & X_{ii} & X_{ii} & \cdots & X_{ii} & X_{ii} \\ * & -\frac{1}{\pi_{i1}^1}X_{i1} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 \\ * & * & -\frac{1}{\pi_{i2}^1}X_{i2} & \cdots & 0 & 0 & \cdots & 0 & 0 \\ * & * & * & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & -\frac{1}{\pi_{i(i-1)}^1}X_{i(i-1)} & 0 & \cdots & 0 & 0 \\ * & * & * & * & * & -\frac{1}{\pi_{i(i+1)}^1}X_{i(i+1)} & \cdots & 0 & 0 \\ * & * & * & * & * & * & \ddots & \vdots & \vdots \\ * & * & * & * & * & * & * & -\frac{1}{\pi_{iN}^1}X_{iN} & 0 \\ * & * & * & * & * & * & * & * & -Q^{-1} \end{bmatrix} \leq 0 \qquad (24)$$

$\hat{\lambda}_2 = 6.7349$, $\bar{\pi}^0 = 100$, $\tilde{\pi}^1 = 0.2$, $\underline{\pi}^1 = 0.2$ and $\underline{\pi}^0 = 80$. Take $\varsigma = 10$, then $M = 125.0134$ and $\tau^* > 0.5624s$. Then, system (22) is 2th moment ISS with average dwell time $\tau^* = 0.6s$.

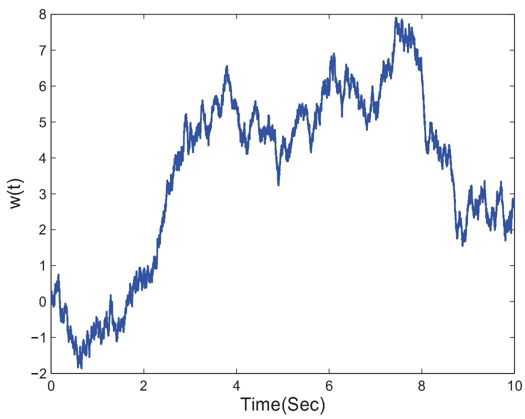

Fig. 2: Brownian motion $w(t)$

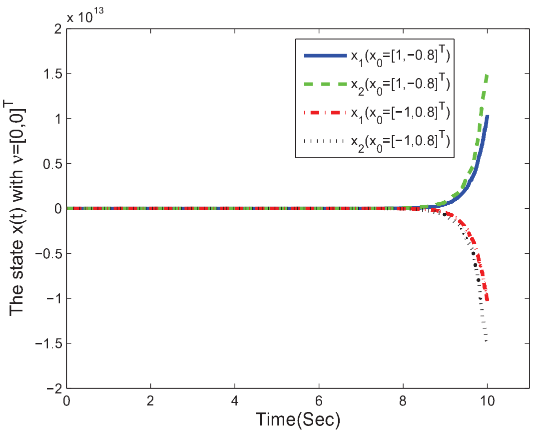

Fig. 3: The state response curves of open-loop system.

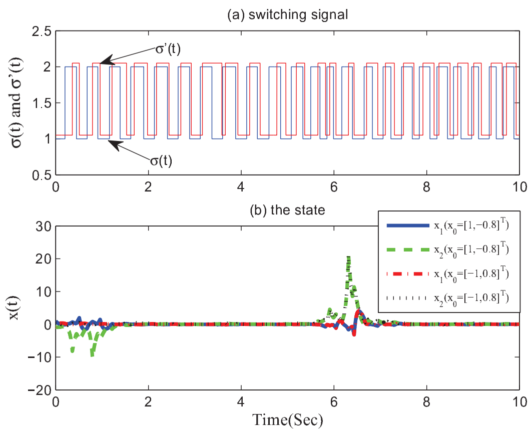

Fig. 4: The state response curves of closed-loop system under normal asynchronous switching controller with $u = [2e^{-0.5t}, 4e^{-0.8t}]^T$.

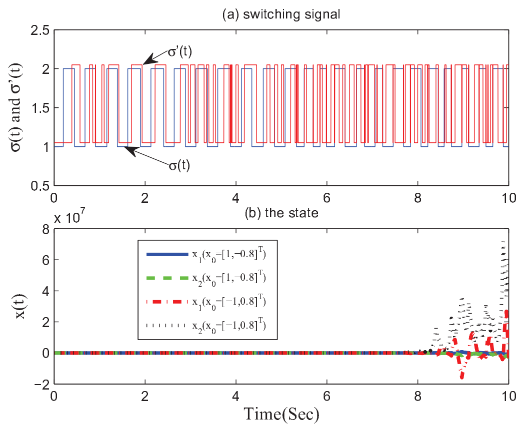

Fig. 5: The state response curves of closed-loop system under extended asynchronous switching controller with $u = [2e^{-0.5t}, 4e^{-0.8t}]^T$.

The first set of simulations are to verify the necessity of performing the research on extended asynchronous switching. The simulation results are shown in Fig.2-Fig.5. Among them, Fig.2 shows the response trajectory of the Brownian motion $w(t)$, while Fig.3 gives the state response curves of open-loop system (21) with the true switching signal given in Fig.4(a), and obviously the open-loop system is unstable. On the other hand, Fig.4(b) and Fig.5(b) present respectively the state trajectories of closed-loop system under normal asynchronous switching controller and extended asynchronous switching controller (note that, it does not satisfy the conditions of Corollary 4.1, because the average dwell time of the true switching is set to be less than $0.5624s$), where Fig.4(a) and Fig.5(a) give the switching signals including the true one and the detected one respectively, with the same true switching signal. Comparing the two results, one can find that the false alarm has a great influence on the control performance, which further verifies the necessity and importance of the extended asynchronous switching system.

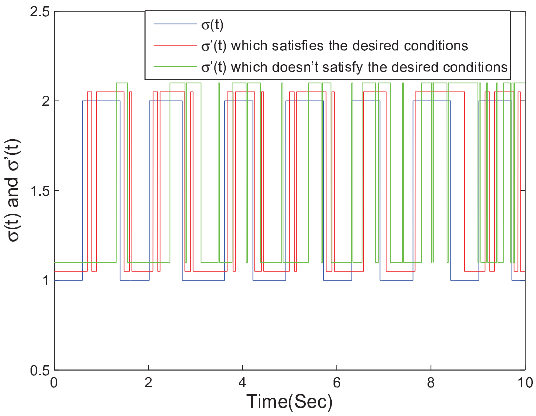

Fig. 6: Switching signal $\sigma(t)$ and the detected $\sigma'(t)$.

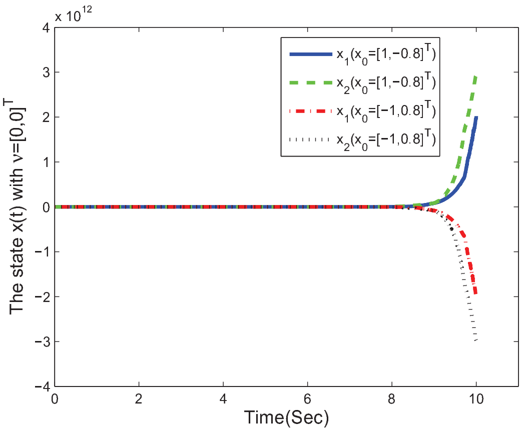

Fig. 7: The open-loop state trajectory.

To demonstrate the effectiveness of the results, the stability under several switching cases are considered, which include the strictly synchronous switching, the desired extended asynchronous switching and the undesired extended asynchronous switching. The simulation results are shown in Fig.6-Fig.16, with the Brownian motion $w(t)$ given in Fig.2. Among them, Fig.6 shows the the true switching signal and the detected switching signal in the presence of detection delay and false alarm, where $\sigma(t)$ is the true switching signal with the desired average dwell time $\tau^* = 0.6$, $\sigma'(t)$ with mode 1.05 and mode 2.05 is the detected switching signal which refers to the desired detected signal (the detection parameters are $\Pi^0 = [-100, 100; 80, -80]$, $\Pi^1 = [-0.2, 0.2; 0.2, -0.2]$, all the conditions in Corollary 4.1 are satisfied), while $\sigma'(t)$ with mode 1.1 and mode 2.1 is the undesired detected one (here, we take $\Pi^0 = \Pi^1 = [-10, 10; 10, -10]$, thus (29) is not satisfied). Note that both the mode 1.05 (mode 2.05) and mode

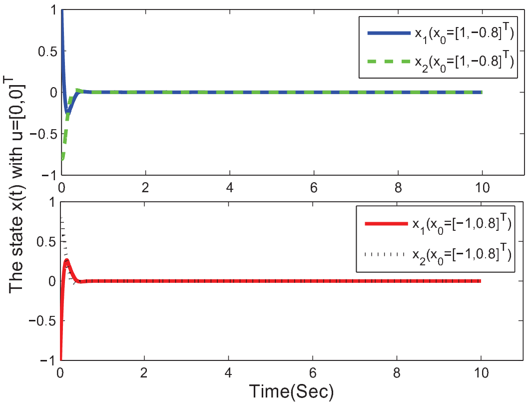

Fig. 8: The closed-loop state trajectory with $u = [0, 0]^T$ under strictly synchronous switching controller.

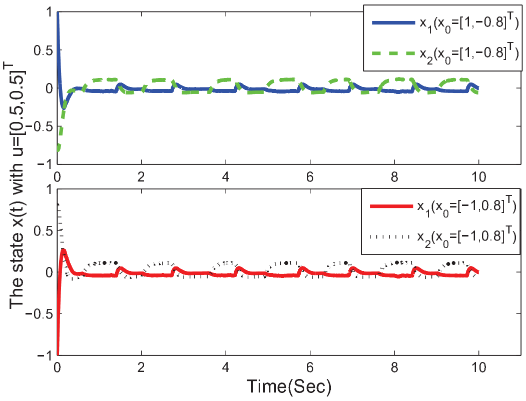

Fig. 9: The closed-loop state trajectory with $u = [0.5, 0.5]^T$ under strictly synchronous switching controller.

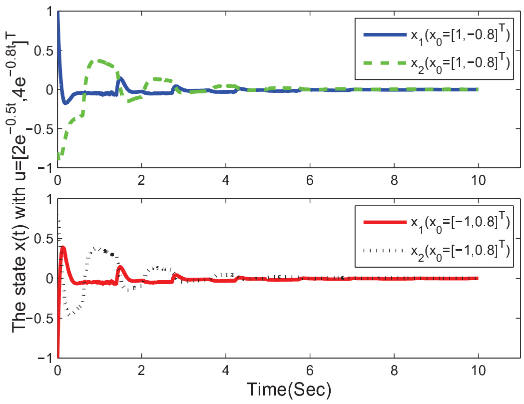

Fig. 10: The closed-loop state trajectory with $u = [2e^{-0.5t}, 4e^{-0.8t}]^T$ under strictly synchronous switching controller.

1.1 (mode 2.1) are referred the mode 1 (mode 2), and these different values are to make clearer illustration. The rest figures show the state trajectories with initial data $x_0 = [\pm 1, \mp 0.8]^T$. Firstly, Fig.7 shows the open-loop state trajectory under the

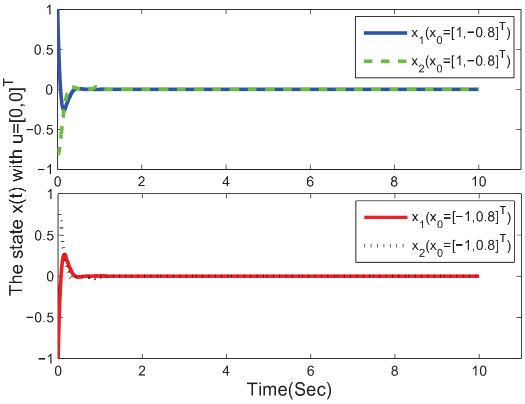

Fig. 11: The closed-loop state trajectory with $u = [0, 0]^T$ under extended asynchronous switching controller which satisfies the desired conditions.

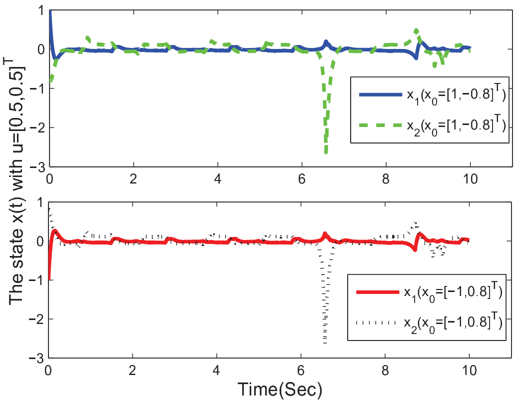

Fig. 12: The closed-loop state trajectory with $u = [0.5, 0.5]^T$ under extended asynchronous switching controller which satisfies the desired conditions.

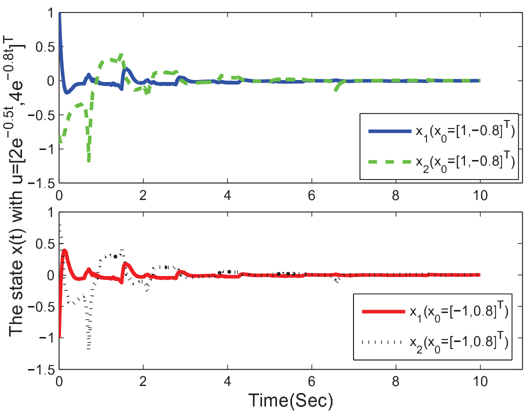

Fig. 13: The closed-loop state trajectory with $u = [2e^{-0.5t}, 4e^{-0.8t}]^T$ under extended asynchronous switching controller which satisfies the desired conditions.

true switching signal $\sigma(t)$ in Fig.6. Obviously, the open-loop system is unstable. Fig.8-Fig.16 show the response curve of the state trajectories when the average dwell time of the true switching signal is the desired one. Among them, Fig.8-Fig.10 show the stability under strictly synchronous switching controller, with reference input $u = [0, 0]^T$, $u = [0.5, 0.5]^T$ and $u = [2e^{-0.5t}, 4e^{-0.8t}]^T$ respectively. Similarly, Fig.11-Fig.13 show the stability under the desired extended asynchronous switching controller, while Fig.14-Fig.16 are performed under the undesired asynchronous switching. From Fig.8-Fig.10, the closed-loop system under strictly synchronous switching controller is stable, in other words, one can claim that the designed controller with considering both detection delay and false alarm (or the designed controller under extended asynchronous controller) are also suitable for the synchronous case. From Fig.11-Fig.13, one can find that the designed controller based on the proposed theory can stabilize the switched system with both non-zero detection delay and false alarm in detection. Compared to Fig.8-Fig.10, one can also find that the asynchronous phenomenon caused by the non-zero detection delay and false alarm has a great impact on the stability. This point can also be further verified by Fig.14-Fig.16. From the results in above three cases, one may claim that the stability of extended asynchronous switching can be guaranteed by a sufficient small mismatched time interval, it is in accordance with Remark 3.3.

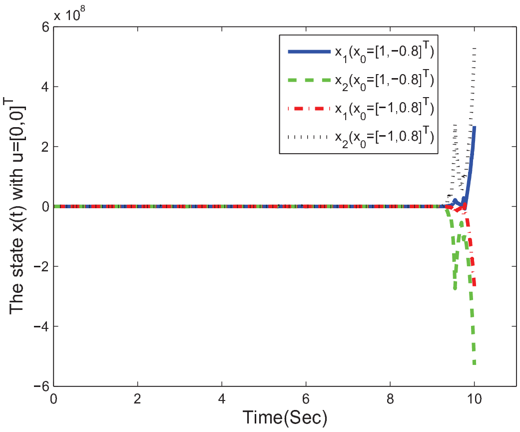

Fig. 14: The closed-loop state trajectory with $u = [0, 0]^T$ under extended asynchronous switching controller which doesn't satisfy the desired conditions.

V. Conclusion

The input-to-state stability of a class of SSNLRS under extended asynchronous switching is investigated. In such systems the switchings of the system modes and the desired mode-dependent controllers are asynchronous due to both detection delays and false alarms, which feature is different from normal asynchronous switching. Through some simplification, an extended asynchronous switching model is developed. Then, based on Razumikhin-type theorem incorporated with average dwell time approach, the sufficient criteria for asymptotic stability as well as input-to-state stability are proposed. It is shown that the stability of such systems can be guaranteed by a sufficient small mismatched time interval and a sufficient large

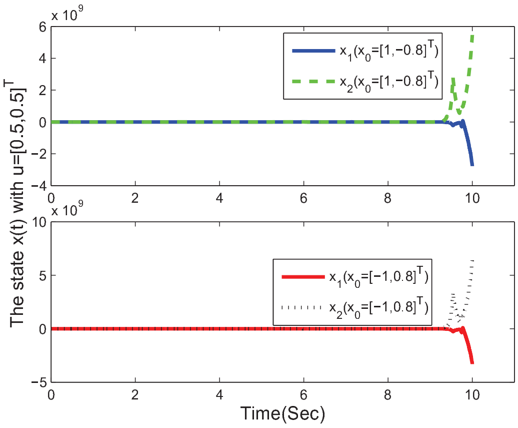

Fig. 15: The closed-loop state trajectory with $u = [0.5, 0.5]^T$ under extended asynchronous switching controller which doesn't satisfy the desired conditions.

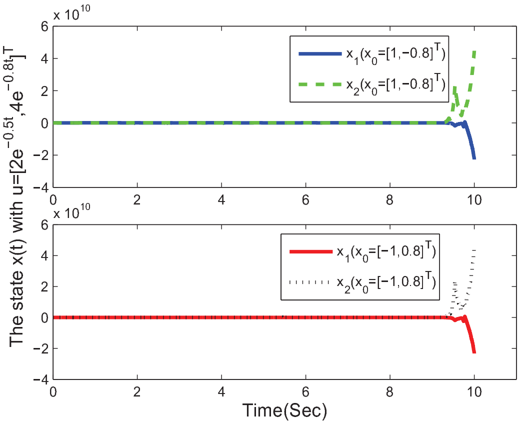

Fig. 16: The closed-loop state trajectory with $u = [2e^{-0.5t}, 4e^{-0.8t}]^T$ under extended asynchronous switching controller which doesn't satisfy the desired conditions.

average dwell time. Finally, the importance and effectiveness of the stability criteria for the extended asynchronous switching system are demonstrated by simulation studies. In the future the developed results are expected to extend to systems with non-exponential distributed detection delays, false alarms, and non-synchronous controller.

Appendix A
Some Lemmas

Lemma A.1: For any given $V(x(t), t, \sigma(t), \sigma'(t)) \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \times \mathcal{S} \times \mathcal{S}; \mathbb{R}_+)$, associated with system (4), the diffusion operator $\mathcal{L}V$, from $C([-\tau, 0]; \mathbb{R}^n) \times \mathbb{R}_+ \times \mathcal{S} \times \mathcal{S}$ to $\mathbb{R}$, can be described as follows.

Case 1. When $\sigma = \sigma' = i$, we have

$$\begin{aligned}\mathcal{L}V(x_t, t, i, i) &= V_t(x, t, i, i) + V_x(x, t, i, i)\bar{f}_{ii}(t, x_t, u)\\&+ \sum_{k=1}^{N} \pi^1_{ik} V(x, t, i, k)\\&+ \frac{1}{2} trace[\bar{g}^T_{ii}(t, x_t, u) V_{xx}(x, t, i, i)\bar{g}_{ii}(t, x_t, u)] \end{aligned}\tag{30}$$

Case 2. When $\sigma' = j$, $\sigma = i$ and $j \neq i$, we also have

$$\begin{aligned}\mathcal{L}V(x_t, t, i, j) &= V_t(x, t, i, j) + V_x(x, t, i, j)\bar{f}_{ij}(t, x_t, u)\\&+ \pi^0_{ji} V(x, t, i, i) - \pi^0_{ji} V(x, t, i, j)\\&+ \frac{1}{2} trace[\bar{g}^T_{ij}(t, x_t, u) V_{xx}(x, t, i, j)\bar{g}_{ij}(t, x_t, u)]\end{aligned}\tag{31}$$

where $i, j \in \mathcal{S}$.

Proof: The proof can be got directly from [33] and [28]. ■

Lemma A.2: Let $V = e^{\lambda t} V(x, t, \sigma, \sigma')$ for all $t \geq 0$ and $\lambda \geq 0$, then

$$\begin{aligned}\mathrm{D}^+\mathbb{E}\{V\} = \mathbb{E}\{\mathcal{L}V\} &= \lambda e^{\lambda t}\mathbb{E}\{V(x, t, \sigma, \sigma')\}\\&+ e^{\lambda t}\mathbb{E}\{\mathcal{L}V(x_t, t, \sigma, \sigma')\}\end{aligned}\tag{32}$$

for all $t \geq 0$, where $\mathrm{D}^+\mathbb{E}\{V\} = \limsup_{dt\to 0^+} \frac{\mathbb{E}\{V(t+dt)\} - \mathbb{E}\{V(t)\}}{dt}$.

Proof: It follows directly from the constructive proof given in [28] (Proof of Theorem 7.3). ■

Lemma A.3: [34] Let $\sigma(t)$ denote a continuous-time Markov process with transition rate matrix $[\pi_{ij}]_{N\times N} \in \mathbb{R}^{N\times N}$, then

$$\mathbb{P}\{N_\sigma(t, 0) = k\} \leq e^{-\tilde{\pi} t}\frac{(\bar{\pi} t)^k}{k!}$$

for any $k \geq 0$, where $\bar{\pi} \triangleq \max_{i\in\mathcal{S}}\{|\pi_{ii}|\}$, $\tilde{\pi} \triangleq \max_{i,j\in\mathcal{S}}\{\pi_{ij}\}$, and $N_\sigma(t, 0)$ denotes the number of switches of $\sigma(t)$ on the time-interval $[0, t]$.

Lemma A.4: For any $i \geq 0$, we have

$$\mathbb{P}\{N(t_{i+1}, t_i) = k\} \leq \begin{cases} e^{-\tilde{\pi}^1(t_{i+1}-t_i)}\frac{(\bar{\pi}^1(t_{i+1}-t_i))^{\frac{k-1}{2}}}{\frac{k-1}{2}!}, \\ \qquad k \text{ is an odd number} \\ e^{-\tilde{\pi}^1(t_{i+1}-t_i)}\frac{(\bar{\pi}^1(t_{i+1}-t_i))^{\frac{k}{2}}}{\frac{k}{2}!}, \\ \qquad k \text{ is an even number}\end{cases}$$

for any $k \in \mathbb{N}_+ \cup \{0\}$.

Proof: Let $N_1(t_{i+1}, t_i)$ denote the numbers of switches from false alarm on time interval $[t_i, t_{i+1})$. In the next, we will complete the proof by considering the following two cases: $N(t_{i+1}, t_i) = 2k + 1$ and $N(t_{i+1}, t_i) = 2k$, where $k \in \mathbb{N}_+ \cup \{0\}$. From the Assumption 2.1, one can obtain $N_1(t_{i+1}, t_i) = \frac{2k+1-1}{2} = k$ in the first case, while $N_1(t_{i+1}, t_i) = \frac{2k}{2} = k$ in the second case. Then, similar to Lemma A.3, it follows

$$\begin{aligned}\mathbb{P}\{N(t_{i+1}, t_i) = 2k + 1\} &\leq \mathbb{P}\{N_1(t_{i+1}, t_i) = \frac{2k + 1 - 1}{2}\}\\&\leq e^{-\tilde{\pi}^1(t_{i+1}-t_i)}\frac{(\bar{\pi}^1(t_{i+1} - t_i))^k}{k!}\end{aligned}$$

and

$$\begin{aligned}\mathbb{P}\{N(t_{i+1}, t_i) = 2k\} &\leq \mathbb{P}\{N_1(t_{i+1}, t_i) = \frac{2k}{2}\}\\&\leq e^{-\tilde{\pi}^1(t_{i+1}-t_i)}\frac{(\bar{\pi}^1(t_{i+1} - t_i))^k}{k!}\end{aligned}$$

Thus we complete the proof. ■

Lemma A.5: For every $i \geq 0$, the moment generating function $\mathbb{E}\{e^{sN(t_{i+1},t_i)}\}$ of $N(t_{i+1}, t_i)$ satisfies

$$\mathbb{E}\{e^{sN(t_{i+1},t_i)}\} \leq (1 + e^s)e^{(e^{2s}\bar{\pi}^1 - \tilde{\pi}^1)(t_{i+1}-t_i)}$$

for any $s \geq 0$.

Proof: Based on Lemma A.4, we have

$$\begin{aligned}
\mathbb{E}\{e^{sN(t_{i+1},t_i)}\} = &\sum_{k=1,3,5,\cdots} e^{sk}\mathbb{P}\{N(t_{i+1},t_i)=k\} \\
&+ \sum_{k=0,2,4,\cdots} e^{sk}\mathbb{P}\{N(t_{i+1},t_i)=k\} \\
\leq &\sum_{k=1,3,5,\cdots} e^{sk}e^{-\bar{\pi}^1(t_{i+1}-t_i)}\frac{(\bar{\pi}^1(t_{i+1}-t_i))^{\frac{k-1}{2}}}{\frac{k-1}{2}!} \\
&+ \sum_{k=0,2,4,\cdots} e^{sk}e^{-\bar{\pi}^1(t_{i+1}-t_i)}\frac{(\bar{\pi}^1(t_{i+1}-t_i))^{\frac{k}{2}}}{\frac{k}{2}!} \\
= &\sum_{k=0,1,2,\cdots} e^{s(2k+1)}e^{-\bar{\pi}^1(t_{i+1}-t_i)}\frac{(\bar{\pi}^1(t_{i+1}-t_i))^{k}}{k!} \\
&+ \sum_{k=0,1,2,\cdots} e^{2sk}e^{-\bar{\pi}^1(t_{i+1}-t_i)}\frac{(\bar{\pi}^1(t_{i+1}-t_i))^{k}}{k!} \\
= &(1+e^s)e^{(e^{2s}\bar{\pi}^1-\bar{\pi}^1)(t_{i+1}-t_i)}
\end{aligned}$$

Thus, we complete the proof. ■

Remark A.1: Following the proof of Lemma A.5, it follows

$$\mathbb{E}\{N(t_{i+1},t_i)\} \leq (1+4\bar{\pi}^1\varsigma)e^{\bar{\pi}^1\varsigma}$$

where $\varsigma = \sup_{l\in\mathbb{N}_+}\{t_l - t_{l-1}\}$.

Appendix B
Proof of Theorem 3.1

Proof: According to (32) in Lemma A.2, it has

$$\mathrm{D}^+\mathbb{E}\{V(x,t,\sigma,\sigma')\} = \mathbb{E}\{\mathcal{L}V(x_t,t,\sigma,\sigma')\} \tag{33}$$

for all $t \in T_s(t_l,t_{l+1}) \cup T_a(t_l,t_{l+1}),\ l \in \mathbb{N}_+$.

On the one hand, from (11), using Jensen's inequality, it holds

$$\begin{aligned}
\mathbb{E}\{V(x,t,i_0,i_0)\} &= \mathbb{E}\{V(x,t,\sigma,\sigma')\} \\
&\leq \mathbb{E}\{\alpha_2(|x|)\} \leq \alpha_2(\mathbb{E}\{\|\xi\|\})
\end{aligned}$$

for any $t \in [t_0-\tau, t_0]$.

In the following, we prove that when $t \in [t_0, t_1)$,

$$\mathbb{E}\{V(x,t,i_0,i_0)\} \leq \alpha_2(\mathbb{E}\{\|\xi\|\})e^{-\bar{\lambda}_1(t-t_0)} \tag{34}$$

Suppose (34) is not true, i.e., there exists some $t \in (t_0, t_1)$ such that

$$\mathbb{E}\{V(x(t),t,i_0,i_0)\} > \alpha_2(\mathbb{E}\{\|\xi\|\})e^{-\bar{\lambda}_1(t-t_0)}.$$

Let $t^* = \inf\{t \in (t_0,t_1) : \mathbb{E}\{V(x(t),t,i_0,i_0)\} > \alpha_2(\mathbb{E}\{\|\xi\|\})e^{-\bar{\lambda}_1(t-t_0)}\}$. By the continuity of $V(x(t),t,i_0,i_0)$ and $x(t)$ on $[t_0,t_1)$, we have $t^* \in [t_0,t_1)$ and $\mathbb{E}\{V(x(t^*),t^*,i_0,i_0)\} = \alpha_2(\mathbb{E}\{\|\xi\|\})e^{-\bar{\lambda}_1(t^*-t_0)}$. Further, there exists a sequence $\{\tilde{t}_n\}$ ($\tilde{t}_n \in (t^*,t_1)$, for any $n \in \mathbb{N}_+$) with $\lim_{n\to\infty}\tilde{t}_n = t^*$, such that

$$\mathbb{E}\{V(x(\tilde{t}_n),\tilde{t}_n),i_0,i_0\} > \alpha_2(\mathbb{E}\{\|\xi\|\})e^{-\bar{\lambda}_1(\tilde{t}_n-t_0)}. \tag{35}$$

Then, from the definition of t^*, we have

$$\begin{aligned}
\mathbb{E}\{V(x(t^*+\theta),t^*+\theta,i_0,i_0)\} &\leq e^{-\bar{\lambda}_1\theta}\mathbb{E}\{V(x(t^*),t^*,i_0,i_0)\} \\
&\leq q\mathbb{E}\{V(x(t^*),t^*,i_0,i_0)\}
\end{aligned}$$

and further,

$$\min_{i,j\in\mathcal{S}}\mathbb{E}\{V(x(t^*+\theta),t^*+\theta,i,j)\} \leq q\mathbb{E}\{V(x(t^*),t^*,i_0,i_0)\}$$

for any $\theta \in [-\tau, 0]$. Then, based on (12) and (13), it holds

$$\mathrm{D}^+\mathbb{E}\{V(x(t^*),t^*,i_0,i_0)\} \leq -\lambda_1\mathbb{E}\{V(x(t^*),t^*,i_0,i_0)\} \tag{36}$$

Without loss of generality, it has

$$\begin{aligned}
\mathrm{D}^+\mathbb{E}\{V(x(t^*),t^*,i_0,i_0)\} &\leq -\lambda_1\mathbb{E}\{V(x(t^*),t^*,i_0,i_0) \\
&< -\bar{\lambda}_1\mathbb{E}\{V(x(t^*),t^*,i_0,i_0)\}
\end{aligned}$$

For $h > 0$ which is sufficient small, when $t \in [t^*, t^*+h]$, it follows

$$\mathrm{D}^+\mathbb{E}\{V(x(t),t,i_0,i_0)\} \leq -\bar{\lambda}_1\mathbb{E}\{V(x(t),t,i_0,i_0)\}$$

which means

$$\mathbb{E}\{V(x(t^*+h),t^*+h,i_0,i_0)\} \leq \mathbb{E}\{V(x(t^*),t^*,i_0,i_0)\}e^{-\bar{\lambda}_1 h}$$

and it is a contradiction of (35). Thus (34) holds.

Combining the continuity of function $V(x(t),t,i_0,i_0)$ and (15), we have

$$\begin{aligned}
\mathbb{E}\{V(x(t_1),t_1,\sigma(t_1),\sigma'(t_1))\} &\leq \mu\mathbb{E}\{V(x(t_1),t_1,i_0,i_0)\} \\
&\leq \mu\alpha_2(\mathbb{E}\{\|\xi\|\})e^{-\bar{\lambda}_1(t_1-t_0)}
\end{aligned} \tag{37}$$

On the other hand, let $W(t,\bar{\sigma}(t)) = W(t,\sigma(t),\sigma'(t)) = e^{\bar{\lambda}_1 t}V(x(t),t,\sigma(t),\sigma'(t))$. Then, for any $l \in \mathbb{N}_+$ and $\theta \in [-\tau, 0]$, we have,

$$\mathrm{D}^+\mathbb{E}\{W(t,\bar{\sigma}(t))\} \leq \begin{cases} -(\lambda_1-\bar{\lambda}_1)\mathbb{E}\{W(t,\bar{\sigma}(t))\}, \\ \qquad t \in T_s(t_l,t_{l+1}) \\ (\bar{\lambda}_1+\lambda_2)\mathbb{E}\{W(t,\bar{\sigma}(t))\}, \\ \qquad t \in T_a(t_l,t_{l+1}) \end{cases}$$

whenever (13) holds.

For any $[s_1,s_2) \subset T_a(t_l,t_{l+1})$, we claim that when $t \in [s_1,s_2)$,

$$\mathbb{E}\{W(t,\bar{\sigma}(t))\} \leq e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t-s_1)}\mathbb{E}\{W(s_1,\bar{\sigma}(s_1))\} \tag{38}$$

Suppose (38) is not true, i.e., there exists some $t \in [s_1,s_2)$ such that

$$\mathbb{E}\{W(t,\bar{\sigma}(t))\} > e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t-s_1)}\mathbb{E}\{W(s_1,\bar{\sigma}(s_1))\}.$$

Similarly, set $t^* = \inf\{t \in [s_1,s_2) : \mathbb{E}\{W(t,\bar{\sigma}(t))\} > \mathbb{E}\{W(s_1,\bar{\sigma}(s_1))\}e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t-s_1)}\}$, then

$$\mathbb{E}\{W(t^*,\bar{\sigma}(t^*))\} = \mathbb{E}\{W(s_1,\bar{\sigma}(s_1))\}e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t^*-s_1)}.$$

Moreover, there is a sequence $\{\tilde{t}_n\}_{n\in\mathbb{N}_+} \in (t^*, s_2)$ with $\lim_{n\to\infty}\tilde{t}_n = t^*$ such that

$$\begin{aligned}
\mathbb{E}\{W(\tilde{t}_n,\bar{\sigma}(\tilde{t}_n))\} &> \mathbb{E}\{W(s_1,\bar{\sigma}(s_1))\}e^{(\bar{\lambda}_1+\bar{\lambda}_2)(\tilde{t}_n-s_1)} \\
&= \mathbb{E}\{W(t^*,\bar{\sigma}(t^*))\}e^{(\bar{\lambda}_1+\bar{\lambda}_2)(\tilde{t}_n-t^*)}
\end{aligned} \tag{39}$$

We further define $U(t) = e^{-(\bar{\lambda}_1+\bar{\lambda}_2)t}W(t,\bar{\sigma}(t))$, then

$$\begin{aligned}
\mathrm{D}^+\mathbb{E}\{U(t)\} = &-\bar{\lambda}_2 e^{-\bar{\lambda}_2 t}\mathbb{E}\{V(x(t),t,\bar{\sigma}(t))\} \\
&+ e^{-\bar{\lambda}_2 t}\mathrm{D}^+\mathbb{E}\{V(x(t),t,\bar{\sigma}(t))\}
\end{aligned}$$

From the definition of t^*, for any $\theta \in [-\tau, 0]$, it follows

$$\begin{aligned}&\mathbb{E}\{W(t^*, \bar{\sigma}(t^*))\}e^{(\bar{\lambda}_1+\bar{\lambda}_2)\theta} \\ &\quad = \mathbb{E}\{W(s_1, \bar{\sigma}(s_1))\}e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t^*+\theta-s_1)} \\ &\quad \geq \mathbb{E}\{W(t^*+\theta, \bar{\sigma}(t^*+\theta))\}\end{aligned}$$

which means

$$\begin{aligned}&\mathbb{E}\{V(x(t^*+\theta), t^*+\theta, \bar{\sigma}(t^*+\theta))\} \\ &\quad \leq \mathbb{E}\{V(x(t^*), t^*, \bar{\sigma}(t^*))\}e^{\bar{\lambda}_2\theta} \leq \mathbb{E}\{V(x(t^*), t^*, \bar{\sigma}(t^*))\}\end{aligned}$$

and further

$$\min_{i,j\in\mathcal{S}} \mathbb{E}\{V(x(t^*+\theta), t^*+\theta, i, j)\} \leq q\mathbb{E}\{V(x(t^*), t^*, \bar{\sigma}(t^*))\}$$

Then, from (12) and (13), we have

$$\mathrm{D}^+\mathbb{E}\{U(t^*)\} \leq -(\bar{\lambda}_2 - \lambda_2)e^{-\bar{\lambda}_2 t^*}\mathbb{E}\{V(x(t^*), t^*, \bar{\sigma}(t^*))\}$$

Without loss of generality, it follows

$$\mathrm{D}^+\mathbb{E}\{U(t^*)\} < 0$$

Moreover, there exists a positive number h which is sufficient small such that

$$\mathrm{D}^+\mathbb{E}\{U(t)\} \leq 0,\ t \in [t^*, t^*+h]$$

It then follows

$$\mathbb{E}\{W(t^*+h, \bar{\sigma}(t^*+h))\} \leq \mathbb{E}\{W(t^*, \bar{\sigma}(t^*))\}e^{(\bar{\lambda}_1+\bar{\lambda}_2)h}$$

which is a contradiction of (39). Thus, (38) is true.

Furthermore, when $t \in [s_1, s_2) \in T_s(t_l, t_{l+1})$, repeating a similar analysis (similar to the proof of (34)), one can obtain

$$\mathbb{E}\{W(t, \bar{\sigma}(t))\} \leq \mathbb{E}\{W(s_1, \bar{\sigma}(s_1))\} \tag{40}$$

Combining (38) and (40), if the detection delay is non-zero, it holds

$$\mathbb{E}\{W(t, \bar{\sigma}(t))\} \leq \begin{cases} e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t-t_l)}\mathbb{E}\{W(t_l, \bar{\sigma}(t_l))\}, & t \in [t_l, t'_{l1}) \\ \mathbb{E}\{W(t'_{l1}, \bar{\sigma}(t'_{l1}))\}, & t \in [t'_{l1}, t'_{l2}) \\ e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t-t'_{l2})}\mathbb{E}\{W(t'_{l2}, \bar{\sigma}(t'_{l2}))\}, & t \in [t'_{l2}, t'_{l3}) \\ \mathbb{E}\{W(t'_{l3}, \bar{\sigma}(t'_{l3}))\}, & t \in [t'_{l3}, t'_{l4}) \\ \cdots \\ e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t-t'_{l(N(t_{l+1},t_l)-1)})} \\ \quad \times \mathbb{E}\{W(t'_{l(N(t_{l+1},t_l)-1)}, \bar{\sigma}(t'_{l(N(t_{l+1},t_l)-1)}))\}, \\ \qquad t \in [t'_{l(N(t_{l+1},t_l)-1)}, t'_{lN(t_{l+1},t_l)}) \\ \mathbb{E}\{W(t'_{lN(t_{l+1},t_l)}, \bar{\sigma}(t'_{lN(t_{l+1},t_l)}))\}, \\ \qquad t \in [t'_{lN(t_{l+1},t_l)}, t_{l+1}) \end{cases} \tag{41}$$

and in this case, $N(t_{l+1}, t_l)$ is an even number. On the other hand, if the detection delay is equal to zero, it also has

$$\mathbb{E}\{W(t, \bar{\sigma}(t))\} \leq \begin{cases} \mathbb{E}\{W(t_l, \bar{\sigma}(t_l))\}, & t \in [t_l, t'_{l1}) \\ e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t-t'_{l1})}\mathbb{E}\{W(t'_{l1}, \bar{\sigma}(t'_{l1}))\}, & t \in [t'_{l1}, t'_{l2}) \\ \mathbb{E}\{W(t'_{l2}, \bar{\sigma}(t'_{l2}))\}, & t \in [t'_{l2}, t'_{l3}) \\ e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t-t'_{l3})}\mathbb{E}\{W(t'_{l3}, \bar{\sigma}(t'_{l3}))\}, & t \in [t'_{l3}, t'_{l4}) \\ \cdots \\ e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t-t'_{l(N(t_{l+1},t_l)-1)})} \\ \quad \times \mathbb{E}\{W(t'_{l(N(t_{l+1},t_l)-1)}, \bar{\sigma}(t'_{l(N(t_{l+1},t_l)-1)}))\}, \\ \qquad t \in [t'_{l(N(t_{l+1},t_l)-1)}, t'_{lN(t_{l+1},t_l)}) \\ \mathbb{E}\{W(t'_{lN(t_{l+1},t_l)}, \bar{\sigma}(t'_{lN(t_{l+1},t_l)}))\}, \\ \qquad t \in [t'_{lN(t_{l+1},t_l)}, t_{l+1}) \end{cases} \tag{42}$$

and in this case, $N(t_{l+1}, t_l)$ is an odd number.

Then, for any $t \in [t_l, t_{l+1})$, if $[t'_{lN(t,t_l)}, t'_{l(N(t,t_l)+1)}) \in T_s(t_l, t_{l+1})$, we can obtain

$$\begin{aligned}&\mathbb{E}\{W(t, \bar{\sigma}(t))\} \leq \mathbb{E}\{W(t'_{lN(t,t_l)}, \bar{\sigma}(t'_{lN(t,t_l)}))\} \\ &\quad \leq \mathbb{E}\{\mu W(t'_{lN(t,t_l)}, \bar{\sigma}(t'_{l(N(t,t_l)-1)}))\} \\ &\quad = \mathbb{E}\{\mu^{N(t,t_l)-N(t,t_l)+1}\}\mathbb{E}\{W(t'_{lN(t,t_l)}, \bar{\sigma}(t'_{l(N(t,t_l)-1)}))\} \\ &\quad \leq \mathbb{E}\{\mu^{N(t,t_l)-N(t,t_l)+1}\}\mathbb{E}\{e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t'_{lN(t,t_l)}-t'_{l(N(t,t_l)-1)})} \\ &\qquad \times W(t'_{l(N(t,t_l)-1)}, \bar{\sigma}(t'_{l(N(t,t_l)-1)}))\} \\ &\quad \leq \mathbb{E}\{\mu^{N(t,t_l)-N(t,t_l)+2}\}\mathbb{E}\{e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t'_{lN(t,t_l)}-t'_{l(N(t,t_l)-1)})}\} \\ &\qquad \times \mathbb{E}\{W(t'_{l(N(t,t_l)-1)}, \bar{\sigma}(t'_{l(N(t,t_l)-2)}))\} \\ &\quad \leq \mathbb{E}\{\mu^{N(t,t_l)-N(t,t_l)+2}\}\mathbb{E}\{e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t'_{lN(t,t_l)}-t'_{l(N(t,t_l)-1)})}\} \\ &\qquad \times \mathbb{E}\{W(t'_{l(N(t,t_l)-2)}, \bar{\sigma}(t'_{l(N(t,t_l)-2)}))\} \\ &\quad \leq \mathbb{E}\{\mu^{N(t,t_l)-N(t,t_l)+3}\}\mathbb{E}\{e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t'_{lN(t,t_l)}-t'_{l(N(t,t_l)-1)})}\} \\ &\qquad \times \mathbb{E}\{W(t'_{l(N(t,t_l)-2)}, \bar{\sigma}(t'_{l(N(t,t_l)-3)}))\} \\ &\quad \leq \mathbb{E}\{\mu^{N(t,t_l)-N(t,t_l)+3}\}\mathbb{E}\{e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t'_{lN(t,t_l)}-t'_{l(N(t,t_l)-1)})} \\ &\qquad \times e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t'_{l(N(t,t_l)-2)}-t'_{l(N(t,t_l)-3)})}\} \\ &\qquad \times \mathbb{E}\{W(t'_{l(N(t,t_l)-3)}, \bar{\sigma}(t'_{l(N(t,t_l)-3)}))\} \\ &\quad \leq \cdots \\ &\quad \leq \mathbb{E}\{\mu^{N(t,t_l)}\}\mathbb{E}\{e^{(\bar{\lambda}_1+\bar{\lambda}_2)T_a(t-t_l)}\}\mathbb{E}\{W(t_l, \bar{\sigma}(t_l))\}.\end{aligned}$$

And similarly, if $[t'_{lN(t,t_l)}, t'_{l(N(t,t_l)+1)}) \in T_a(t_l, t_{l+1})$, it also follows that

$$\begin{aligned}&\mathbb{E}\{W(t, \bar{\sigma}(t))\} \\ &\quad \leq \mathbb{E}\{\mu^{N(t,t_l)}\}\mathbb{E}\{e^{(\bar{\lambda}_1+\bar{\lambda}_2)T_a(t-t_l)}\}\mathbb{E}\{W(t_l, \bar{\sigma}(t_l))\}\end{aligned}$$

Then, without loss of generality, for any $t \in [t_l, t_{l+1})$, it holds

$$\begin{aligned}&\mathbb{E}\{W(t, \bar{\sigma}(t))\} \\ &\quad \leq \mathbb{E}\{\mu^{N(t,t_l)}\}\mathbb{E}\{e^{(\bar{\lambda}_1+\bar{\lambda}_2)T_a(t-t_l)}\}\mathbb{E}\{W(t_l, \bar{\sigma}(t_l))\} \\ &\quad \leq \mathbb{E}\{\mu^{N(t_{l+1},t_l)}\}\mathbb{E}\{e^{(\bar{\lambda}_1+\bar{\lambda}_2)T_a(t_{l+1}-t_l)}\}\mathbb{E}\{W(t_l, \bar{\sigma}(t_l))\}\end{aligned} \tag{43}$$

On the other hand, for any $l \geq 0$, it holds (44a) and (44b).

$$\mathbb{E}\{e^{(\bar{\lambda}_1+\bar{\lambda}_2)T_a(t_{l+1}-t_l)}\} = \begin{cases} \mathbb{E}\{\underbrace{e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t'_{l1}-t'_{l0})}e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t'_{l3}-t'_{l2})}\dots e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t'_{lN(t_{l+1},t_l)}-t'_{l(N(t_{l+1},t_l)-1)})}}_{\frac{N(t_{l+1},t_l)+1}{2},\ N(t_{l+1},t_l)\ is\ an\ odd\ number}\} & (44a) \\ \mathbb{E}\{\underbrace{e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t'_{l2}-t'_{l1})}e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t'_{l4}-t'_{l3})}\dots e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t'_{lN(t_{l+1},t_l)}-t'_{l(N(t_{l+1},t_l)-1)})}}_{\frac{N(t_{l+1},t_l)}{2},\ N(t_{l+1},t_l)\ is\ an\ even\ number}\} & (44b) \end{cases}$$

Since

$$\mathbb{E}\{e^{(\bar{\lambda}_1+\bar{\lambda}_2)(t'_{lj}-t'_{lj-1})}\} \leq \int_0^\infty e^{(\bar{\lambda}_1+\bar{\lambda}_2)t}\bar{\pi}^0 e^{-\underline{\pi}^0 t}\mathrm{d}t = \frac{-\bar{\pi}^0}{\bar{\lambda}_1+\bar{\lambda}_2-\underline{\pi}^0}$$

then, based on Lemma A.5, let $s = \ln\sqrt{\frac{-\bar{\pi}^0}{\bar{\lambda}_1+\bar{\lambda}_2-\underline{\pi}^0}}$, we have

$$\begin{aligned} &\mathbb{E}\{e^{(\bar{\lambda}_1+\bar{\lambda}_2)T_a(t_{l+1}-t_l)}\} \\ &\leq \begin{cases} \mathbb{E}\{(\frac{-\bar{\pi}^0}{\bar{\lambda}_1+\bar{\lambda}_2-\underline{\pi}^0})^{\frac{N(t_{l+1},t_l)+1}{2}}\}, \\ \quad N(t_{l+1},t_l) \text{ is an odd number} \\ \mathbb{E}\{(\frac{-\bar{\pi}^0}{\bar{\lambda}_1+\bar{\lambda}_2-\underline{\pi}^0})^{\frac{N(t_{l+1},t_l)}{2}}\}, \\ \quad N(t_{l+1},t_l) \text{ is an even number} \end{cases} \\ &\leq \begin{cases} (\sqrt{K}_1+K_1)e^{(K_1\bar{\pi}^1-\tilde{\pi}^1)(t_{l+1}-t_l)}, \\ \quad N(t_{l+1},t_l) \text{ is an odd number} \\ (1+\sqrt{K}_1)e^{(K_1\bar{\pi}^1-\tilde{\pi}^1)(t_{l+1}-t_l)}, \\ \quad N(t_{l+1},t_l) \text{ is an even number} \end{cases} \end{aligned} \tag{45}$$

and without loss of generality,

$$\mathbb{E}\{e^{(\bar{\lambda}_1+\bar{\lambda}_2)T_a(t_{l+1}-t_l)}\} \leq K_2 e^{(K_1\bar{\pi}^1-\tilde{\pi}^1)(t_{l+1}-t_l)}$$

where $K_1 = \frac{-\bar{\pi}^0}{\bar{\lambda}_1+\bar{\lambda}_2-\underline{\pi}^0}$, $K_2 = K_1+\sqrt{K}_1 = \max\{\sqrt{K}_1+K_1, 1+\sqrt{K}_1\}$.

In addition, if we let $s = \ln(\mu)$, utilizing the Lemma A.5 again, we can obtain

$$\mathbb{E}\{\mu^{N(t_{l+1},t_l)}\} \leq (1+\mu)e^{(\mu^2\bar{\pi}^1-\tilde{\pi}^1)(t_{l+1}-t_l)}.$$

Consequently, for any $t \in [t_l, t_l+1)$, it has

$$\begin{aligned} \mathbb{E}\{W(t,\bar{\sigma}(t))\} &\leq K_3 e^{k_1(t_{l+1}-t_l)}\mathbb{E}\{W(t_l,\bar{\sigma}(t_l))\} \\ &\leq K_3 e^{\bar{k}_1(t_{l+1}-t_l)}\mathbb{E}\{W(t_l,\bar{\sigma}(t_l))\} \\ &\leq M\mathbb{E}\{W(t_l,\bar{\sigma}(t_l))\} \end{aligned} \tag{46}$$

where $K_3 = (1+\mu)K_2$, $k_1 = (\mu^2+K_1)\bar{\pi}^1 - 2\tilde{\pi}^1$, $k_1 = [(\mu^2+K_1)(N-1)-2]\tilde{\pi}^1$, $M = K_3 e^{\bar{k}_1\varsigma}$.

From (15), for any $t \geq t_1$, iterating (46) from $l = 1$ to $l = N_\sigma(t,t_1)+1$, we can get

$$\begin{aligned} \mathbb{E}\{W(t,\bar{\sigma}(t))\} &\leq M\mathbb{E}\{W(t_{N_\sigma(t,t_1)+1},\bar{\sigma}(t_{N_\sigma(t,t_1)+1}))\} \\ &\leq \mu M^2\mathbb{E}\{W(t_{N_\sigma(t,t_1)},\bar{\sigma}(t_{N_\sigma(t,t_1)}))\} \\ &\leq \mu^2 M^3\mathbb{E}\{W(t_{N_\sigma(t,t_1)-1},\bar{\sigma}(t_{N_\sigma(t,t_1)-1}))\} \\ &\leq \cdots \\ &\leq \mu^{N_\sigma(t,t_1)}M^{N_\sigma(t,t_1)+1}\mathbb{E}\{W(t_1,\bar{\sigma}(t_1))\} \end{aligned}$$

which means for any $t \geq t_1$,

$$\begin{aligned} \mathbb{E}\{V(x(t),t,\sigma(t),\sigma'(t))\} &\leq \mu^{N_\sigma(t,t_1)}M^{N_\sigma(t,t_1)+1}e^{-\bar{\lambda}_1(t-t_1)} \\ &\quad \times \mathbb{E}\{V(x(t_1),t_1,\sigma(t_1),\sigma'(t_1))\} \end{aligned} \tag{47}$$

Combining (37) and (47), we have

$$\begin{aligned} &\mathbb{E}\{V(x(t),t,\sigma(t),\sigma'(t))\} \\ &\leq \mu^{N_\sigma(t,t_1)+1}M^{N_\sigma(t,t_1)+1}e^{-\bar{\lambda}_1(t-t_0)}\alpha_2(\mathbb{E}\{\|\xi\|\}) \\ &= (\mu M)^{N_\sigma(t,t_0)}e^{-\bar{\lambda}_1(t-t_0)}\alpha_2(\mathbb{E}\{\|\xi\|\}) \\ &\leq (\mu M)^{N_0}e^{(-\bar{\lambda}_1+\frac{\ln(\mu M)}{\tau^*})(t-t_0)}\alpha_2(\mathbb{E}\{\|\xi\|\}) \\ &\triangleq \tilde{\beta}(\mathbb{E}\{\|\xi\|\}, t-t_0) \end{aligned} \tag{48}$$

for any $t \geq t_1$. Clearly, $\tilde{\beta}(\cdot,\cdot) \in \mathcal{KL}$ if and only if $\tau^* > \frac{\ln(\mu M)}{\bar{\lambda}_1}$. For any $\varepsilon \in (0,1)$, take $\bar{\beta} = \frac{\tilde{\beta}}{\varepsilon} \in \mathcal{KL}$. Obviously, (48) also holds for $t \in [t_0, t_1)$. Then, using Chebyshev's inequality and the above inequality, for all $t \geq t_0$,

$$\begin{aligned} &\mathbb{P}\{V(x(t),t,\sigma(t),\sigma'(t)) \geq \bar{\beta}(\mathbb{E}\{\|\xi\|\}, t-t_0)\} \\ &\quad \leq \frac{\mathbb{E}\{V(x(t),t,\sigma(t),\sigma'(t))\}}{\bar{\beta}(\mathbb{E}\{\|\xi\|\}, t-t_0)} < \varepsilon \end{aligned}$$

Define $\beta(r,s) = \alpha_1^{-1}\circ\bar{\beta}(r,s)$, then

$$\mathbb{P}\{|x(t)| < \beta(\mathbb{E}\{\|\xi\|\}, t-t_0)\} \geq 1-\varepsilon,\ \forall t \geq t_0$$

where $\beta(\cdot,\cdot) \in \mathcal{KL}$. Thus, we complete the proof. ■

Appendix C
Proof of Corollary 4.1

Proof: Take $V(x(t),t,i,j) = x^T(t)P_{ij}x(t)$, $P_{ij} = P_{ij}^T > 0$, for any $i,j \in \mathcal{S}$. We assume that there exist $\beta_1 > 0$ and $\beta_2 > 0$ such that $P_{ii} < \beta_1 I$ and $P_{ij} < \beta_2 I$, where I is an identity matrix with appropriate dimension.

When $t \in T_s(t_l, t_{l+1})$, the system in (22) can be written as

$$\mathrm{d}x = [(A_i+B_iK_i)x + B_i u]\mathrm{d}t + g(t,x(t-d),i)\mathrm{d}w. \tag{49}$$

Then,

$$\begin{aligned} &\mathcal{L}V(x,y,t,i,i) \\ &= 2x^T P_{ii}[(A_i+B_iK_i)x + B_i u] \\ &\quad + \frac{1}{2}g^T(t,y,i)P_{ii}g(t,y,i) + \sum_{k=1}^N \pi_{ik}^1 x^T P_{ik}x \\ &\leq x^T[(A_i+B_iK_i)^T P_{ii} + P_{ii}(A_i+B_iK_i) \\ &\quad + \varepsilon_1 P_{ii}B_iB_i^T P_{ii} + \sum_{k=1}^N \pi_{ik}^1 P_{ik}]x + \varepsilon_1^{-1}u^T u \\ &\quad + \frac{1}{2}\beta_1 g^T(t,y,i)g(t,y,i) \\ &\leq x^T[(A_i+B_iK_i)^T P_{ii} + P_{ii}(A_i+B_iK_i) + \varepsilon_1^{-1}\|u\|_2^2 \\ &\quad + \varepsilon_1 P_{ii}B_iB_i^T P_{ii} + \sum_{k=1}^N \pi_{ik}^1 P_{ik}]x + \frac{1}{2}\beta_1 y^T U_i^T U_i y \end{aligned}$$

for some $\varepsilon_1 > 0$, where $y(t) = x(t - d(t))$, by considering the fact that $HFE + E^T F^T H^T \leq \varepsilon HH^T + \varepsilon^{-1} E^T E$ where $\varepsilon > 0$, $FF^T \leq I$.

When $t \in T_a(t_l, t_{l+1})$, the system in (22) can be written as

$$\mathrm{d}x = [(A_i + B_i K_j)x + B_i u]\mathrm{d}t + g(t, x(t-d), i)\mathrm{d}w \quad (50)$$

where $i, j \in \mathcal{S}$, and $i \neq j$. Similarly, we have

$$\begin{aligned}\mathcal{L}V(x, y, t, i, j) \leq\ & x^T[(A_i + B_i K_j)^T P_{ij} + P_{ij}(A_i + B_i K_j) \\ & + \varepsilon_2 P_{ij} B_i B_i^T P_{ij} + \pi_{ji}^0 (P_{ii} - P_{ij})]x \\ & + \frac{1}{2}\beta_2 y^T U_i^T U_i y + \varepsilon_2^{-1}\|u\|_2^2\end{aligned}$$

for some $\varepsilon_2 > 0$.

For any non-negative definite matrix Q, we have

$$|x| \geq \sqrt{\frac{1}{\varepsilon\lambda_{min}(Q)}}\|u\|_2 \Rightarrow$$
$$\begin{cases} \mathcal{L}V(x, y, t, i, i) \leq x^T \Phi_{ii} x + \frac{1}{2}\beta_1 y^T U_i^T U_i y \\ \mathcal{L}V(x, y, t, i, j) \leq x^T \Phi_{ij} x + \frac{1}{2}\beta_2 y^T U_i^T U_i y \end{cases}$$

where $\varepsilon = \min\{\varepsilon_1, \varepsilon_2\}$, $\lambda_{min}(Q)$ denotes the minimal eigenvalue of matrix Q, and

$$\begin{aligned}\Phi_{ii} =\ & (A_i + B_i K_i)^T P_{ii} + P_{ii}(A_i + B_i K_i) \\ & + \varepsilon_1 P_{ii} B_i B_i^T P_{ii} + \sum_{k=1}^{N} \pi_{ik}^1 P_{ik} + Q, \\ \Phi_{ij} =\ & (A_i + B_i K_j)^T P_{ij} + P_{ij}(A_i + B_i K_j) \\ & + \varepsilon_2 P_{ij} B_i B_i^T P_{ij} + \pi_{ji}^0 (P_{ii} - P_{ij}) + Q\end{aligned}$$

Further, if

$$\begin{aligned}\mathcal{L}V(x, y, t, i, i) &\leq x^T \Phi_{ii} x + \frac{1}{2}\beta_1 y^T U_i^T U_i y \\ &\leq -\lambda_1 x^T P_{ii} x + \lambda_{10} y^T P_{ii} y \quad (51)\end{aligned}$$

and

$$\begin{aligned}\mathcal{L}V(x, y, t, i, j) &\leq x^T \Phi_{ij} x + \frac{1}{2}\beta_2 y^T U_i^T U_i y \\ &\leq \lambda_2 x^T P_{ij} x + \lambda_{20} y^T P_{ij} y \quad (52)\end{aligned}$$

and (15) and (19) hold, then based on Corollary 3.1, the conclusion is obtained.

On the other hand, the conditions in (51) and (52) can be transformed into

$$\begin{aligned}&(A_i + B_i K_i)^T P_{ii} + P_{ii}(A_i + B_i K_i) + \varepsilon_1 P_{ii} B_i B_i^T P_{ii} \\ &\qquad + \sum_{k=1}^{N} \pi_{ik}^1 P_{ik} + Q + \lambda_1 P_{ii} \leq 0 \quad (53) \\ &\frac{1}{2}\beta_1 U_i^T U_i - \lambda_{10} P_{ii} \leq 0 \quad (54) \\ &(A_i + B_i K_j)^T P_{ij} + P_{ij}(A_i + B_i K_j) + \varepsilon_2 P_{ij} B_i B_i^T P_{ij} \\ &\qquad + \pi_{ji}^0 (P_{ii} - P_{ij}) + Q - \lambda_2 P_{ij} \leq 0 \quad (55) \\ &\frac{1}{2}\beta_2 U_i^T U_i - \lambda_{20} P_{ij} \leq 0 \quad (56)\end{aligned}$$

where $i, j \in \mathcal{S}$ and $i \neq j$. Using P_{ii}^{-1} to pre- and post-multiply the left term of equation (53) and (54) respectively and denoting $X_{ii} = P_{ii}^{-1}$, $X_{ij} = P_{ij}^{-1}$, $Y_{ii} = K_i X_{ii}$ and $Y_{ij} = K_j X_{ij}$ yields (24) and (25).

Similarly, using P_{ij}^{-1} to pre- and post- multiply the left term of equation (55) and (56) respectively yields (26) and (27). It is easy to get that (51) and (52) hold, if the LMIs (24)-(27) hold. By taking proper λ_i and λ_{i0}, $i = 1, 2$, then there exists q such that (28) and (29) hold. And further, by solving (24)-(27) and (15), we can get the control gains K_i, $i \in \mathcal{S}$. ■

REFERENCES

[1] D. Liberzon, *Switching in systems and control*, Boston, MA:Birkhauser, 2003.

[2] Z. D. Sun, S. S. Ge, *Stability theory of switched dynamical systems*, London:Springer-Verlag, 2011.

[3] M. S. Mahmoud, P. Shi, *Methodologies for control of jumping time-delay systems*, Kluwer Academic Publishers, Amsterdam, 2003.

[4] B. Lu, F. Wu, S. Kim, "Switching LPV control of an F-16 aircraft via controller state reset," *IEEE Trans. Contr. Syst. T.*, vol.14, no.2, pp. 267-277, 2006.

[5] P. Sun, S. Y. Wang, "Redundant input guaranteed cost switched tracking control for omnidirectional rehabilitative training walker," *Int. J. Innov. Comput. I. Contr.*, vol.10, no.3, pp.883-895, 2014.

[6] Y. Q. Xia, L. Li, G. P. Liu, P. Shi, "H_∞ predictive control of networked control systems," *Int. J. Control*, vol.84, no.6, pp.1080-1097, 2011.

[7] S. S. Ge, Z. D. Sun, T. H. Lee, "Reachability and controllability of switched linear discrete-time systems," *IEEE Trans. Autom. Contr.*, vol.46, no.9, pp. 1437-1441, 2001.

[8] J. Lian, P. Shi, Z. Feng, "Passivity and passification for a class of uncertain switched stochastic time-delay systems," *IEEE Trans. Cybern.*, vol.43, no.1, pp.3-13, 2013.

[9] M. S. Mahmoud, "Delay-dependent H_∞ filtering of a class of switched discrete-time state delay systems," *Signal Process.*, vol.88, no.11, pp.2709-2719, 2008.

[10] X. D. Zhao, L. X. Zhang, P. Shi, M. Liu, "Stability of switched positive linear systems with average dwell time switching," *Automatica*, vol.48, no.6, pp.1132-1137, 2012.

[11] M. S. Mahmoud, "Switched discrete-time with time-varying delays: a generalized H_2 approach," *Comput. Math. Appl.*, vol.57, no.1, pp.79-95, 2009.

[12] X. D. Zhao, L. X. Zhang, P. Shi, M. Liu, "Stability and stabilization of switched linear systems with mode-dependent average dwell time," *IEEE Trans. Autom. Contr.*, vol.57, no.7, pp. 1809-1815, 2012.

[13] V. Veselý, D. Rosinová, "Robust MPC controller design for switched systems using multi-parameter dependent lyapunov function," *Int. J. Innov. Comput. I. Contr.*, vol.10, no.1, pp.269-280, 2014.

[14] Y. Q. Xia, M. Y. Fu, G.-P. Liu, *Analysis and synthesis of networked control systems*, Springer, 2011.

[15] Z. R. Xiang, R. H. Wang, Q. W. Chen, "Robust reliable stabilization of stochastic switched nonlinear systems under asynchronous switching," *Appl. Math. Comput.*, vol.217, pp. 7725-7736, 2011.

[16] Z. R. Xiang, C. H. Qiao, M. S. Mahmoud, "Robust H_∞ filtering for switched stochastic systems under asynchronous switching," *J. Franklin I.*, vol.349, no.3, pp. 1213-1230, 2012.

[17] Z. R. Xiang, R. H. Wang, Q. W. Chen, "Robust stabilization of uncertain stochastic switched non-linear systems under asynchronous switching," *P. I. Mech. Eng. I.-J. Sys.*, vol.225, no.1, pp. 8-20, 2011.

[18] L. X. Zhang, H. J. Gao, "Asynchronously switched control of swicthed linear systems with average dwell time," *Automatica*, vol.46, pp. 953-958, 2010.

[19] L. X. Zhang, P. Shi, "Stability, l_2-gain and asynchronous H_∞ control of discrete-time switched systems with average dwell time," *IEEE Trans. Autom. Contr.*, vol.54, no.9, pp. 2193-2200, 2009.

[20] L. Vu, K. A. Morgansen, "Stability of time-delay feedback switched linear systems," *IEEE Trans. Autom. Contr.*, vol.55, no.10, pp. 2385-2390, 2010.

[21] M. S. Mahmoud, *Switched time-delay systems: stability and control*, New York:Springer, 2010.

[22] B. Z. Wu, P. Shi, H. Su, J. Chu, "Delay-dependent stability analysis for switched neural networks with time-varying delay," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol.41, no.6, pp. 1522-1530, 2011.

[23] W. X. Xie, C. Y. Wen, Z. G. Li, "Input-to-state stabilization of switched nonlinear systems," *IEEE Trans. Autom. Contr.*, vol.46, no.7, pp. 1111-1116, 2001.

[24] Z. R. Xiang, R. H. Wang, Q. W. Chen, "Fault tolerant control of switched nonlinear systems with time delay under asynchronous switching," *Int. J. Appl. Math. Comput. Sci.*, vol.20, no.3, pp. 497-506, 2010.
[25] Y. E. Wang, X. M. Sun, P. Shi, "Input-to-state stability of switched nonlinear systems with time delays under asynchronous switching," *IEEE Trans. Cybern.*, vol.43, no.6, pp.2261-2265, 2013.
[26] M. S. Mahmoud, P. Shi, "Asynchronous H_∞ filtering of discrete-time switched systems," *Signal Process.*, vol.92, no.10, pp.2356-2364, 2012.
[27] Y. Kang, D. H. Zhai, G. P. Liu, Y. B. Zhao, P. Zhao, "Stability analysis of a class of hybrid stochastic retarded systems under asynchronous switching," *IEEE Trans. Autom. Contr.*, vol.49, no.6, pp.1511-1523, 2014.
[28] M. Marition, *Jump linear systems in automatic control*, Marcel Dekker, NewYork, 1990.
[29] Y. Kang, J. F. Zhang, S. S. Ge, "Robust output feedback H_∞ control of uncertain Markovian jump systems with mode-dependent time-delays," *Int. J. Control*, vol.81, no.1, pp. 43-61, 2008.
[30] H. K. Khalil, *Nonlinear systems, 3rd Edition*, Prentice-Hall, Englewood Cliffs New Jersey, 2002.
[31] L. R. Huang, X. R. Mao, "On input-to-state stability of stochastic retarded systems with Markovian switching," *IEEE Trans. Autom. Contr.*, vol.54, no.8, pp. 1898-1902, 2009.
[32] S. J. Liu, J. F. Zhang, Z. P. Jiang, "A notion of stochastic input-to-state stability and its application to stability of cascaded stochastic nonlinear systems," *Acta Math. Appl. Sin.-E.*, vol.24, no.1, pp. 141-156, 2008.
[33] X. R. Mao, "Razumikhin-type theorems on exponential stability of stochastic functional differential equations," *Stoch. Proc. Appl.*, vol.65, pp. 233-250, 1996.
[34] D. Chatterjee, D. Liberzon, "On stability of randomly switched nonlinear systems," *IEEE Trans. Autom. Contr.*, vol.52, no.12, pp. 2390-2394, 2007.
[35] X. T. Wu, L. T. Yan, W. B. Zhang, Y. Tang, "Stability of stochastic nonlinear switched systems with average dwell time," *J. Phys. A: Math. Theor.*, vol.45, no.8, pp. 5207-5217, 2012.

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