

Design and Performance Analysis of Incremental Networked Predictive Control Systems

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Abstract—This paper is concerned with the design and performance analysis of networked control systems (NCSs) with network-induced delay, packet disorder, and packet dropout. Based on the incremental form of the plant input-output model and an incremental error feedback control strategy, an incremental networked predictive control (INPC) scheme is proposed to actively compensate for the round-trip time delay resulting from the above communication constraints. The output tracking performance and closed-loop stability of the resulting INPC system are considered for two cases: plant-model match case and plant-model mismatch case. For the former case, the INPC system can achieve the same output tracking performance and closed-loop stability as those of the corresponding local control system. For the latter case, a sufficient condition for the stability of the closed-loop INPC system is derived using the switched system theory. Furthermore, for both cases, the INPC system can achieve a zero steady-state output tracking error for step commands. Finally, both numerical simulations and practical experiments on an Internet-based servo motor system illustrate the effectiveness of the proposed method.

Index Terms—Networked control systems (NCSs), predictive control, round-trip time delay, performance analysis, stability analysis, experiment.

I. INTRODUCTION

NETWORKED control systems (NCSs) are control systems which are closed via communication networks. The introduction of networks into the control loop inevitably causes some adverse effects such as network-induced delay, packet disorder, and packet dropout, usually resulting in performance degradation or even instability of NCSs. Therefore, various approaches have been presented to cope with them, such as stochastic system approach, switched system approach, time delay system approach, and robust control approach [1]. However, the aforementioned approaches have not taken full advantage of the specialities of NCSs, for example, the packet-based transmission, timestamp technique, and smart sensors and actuators. A typical approach making full use of them is networked predictive control (NPC) methodology. For instance, in [2]-[9], some NPC methods have been proposed based on state-space model, mainly on the issues of stability analysis and stabilization control.

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Tracking control is a fundamentally important task in practical applications like industry manipulators, motor systems, mobile robots, autonomous vehicles, aircraft systems, and so on [10]-[13]. Compared with the stabilization problem, it is generally more challenging [14], [15], and only a few studies are carried out especially for the tracking control problems of NCSs [16]-[19]. Based on the state-space model, NPC methods were proposed to deal with the output tracking problem of NCSs with Markov network-induced delay in [15] and [20]. However, the system state needs to be online measured, and the priori occurrence probability of the network-induced delay must be known, which are usually not available in practice.

As an alternative, in [21]-[27], NPC methods were developed based on the input-output difference equation model, which is easy to implement in practice. However, there exist some common drawbacks in these works: i) Since these NPC methods were designed based on the original model of the plant, they would cause a steady-state output tracking error for a step reference signal due to the inevitable plant-model mismatch in practice. ii) In aforementioned works, only some sufficient conditions for the closed-loop stability were obtained, which have considerable conservativeness. Furthermore, only the plant-model match case of NCSs was considered in [21]-[26]. iii) The output tracking performance was not theoretically investigated in [21]-[26], which is especially important for the practical applications of these NPC methods. The foregoing three facts motivate the present study.

In this paper, to overcome the above deficiencies of existing NPC methods in [21]-[27], a modified NPC scheme is proposed for the output tracking control of networked systems, which is designed based on the incremental form of plant model and an incremental error feedback control law. Therefore, it is called incremental networked predictive control (INPC). The communication constraints such as network-induced delay, packet disorder, and packet dropout in the feedback and forward channels are considered and treated as the random round-trip time (RTT) delay. The main contributions of this paper include the following two aspects: 1) an incremental NPC method is presented to actively compensate for the RTT delay, and 2) the output tracking performance and closed-loop stability of the resulting INPC system are investigated for both the plant-model match and mismatch cases. Furthermore, some comparisons between the INPC method and the existing NPC methods are provided.

The remainder of this paper is organized as follows. Section II presents an INPC scheme. The main results of performance and stability are given in Section III. In Section IV and V, the proposed method is validated via simulations and experiments,

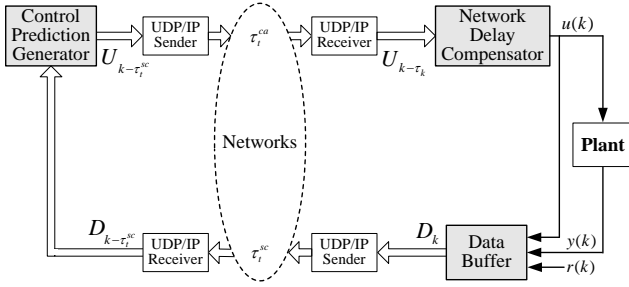


Fig. 1. INPC systems.

respectively. Section VI concludes this paper.

Notation: The notations used throughout the paper are fairly standard. Scalars, vectors, and matrices are denoted by lowercase letters, uppercase letters, and uppercase bold letters, respectively. \mathbf{I}_n and $\mathbf{0}_{n \times m}$ denote an n -dimensional identity matrix and an $n \times m$ zero matrix, respectively. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. Δ represents the difference operator, i.e., $\Delta = 1 - z^{-1}$.

II. INPC SCHEME

Consider a linear plant described by

$$a(z^{-1})y(k) = b(z^{-1})u(k-1) \quad (1)$$

where $y(k) \in \mathbb{R}$ and $u(k) \in \mathbb{R}$ are the output and input of the plant at sampling instant k , respectively. $a(z^{-1})$ and $b(z^{-1})$ are the polynomials with the orders of n_a and n_b , respectively, as follows:

$$\begin{cases} a(z^{-1}) = 1 + a_1 z^{-1} + \dots + a_{n_a} z^{-n_a} \\ b(z^{-1}) = b_0 + b_1 z^{-1} + \dots + b_{n_b} z^{-n_b} \end{cases}$$

For the local control of system (1), i.e., there exist no networks between the controller and the plant, an incremental controller can be designed as

$$\Delta u(k) = KE(k) \quad (2)$$

where $K \in \mathbb{R}^{n_e+1}$ is the controller gain to be determined, and $E(k) = [e(k) \ e(k-1) \ \dots \ e(k-n_e)]^T$ with the output tracking error

$$e(k) = r(k) - y(k) \quad (3)$$

where $r(k)$ is a reference signal. From (1) and (3), we have

$$\Delta y(k+1) = (1 - a(z^{-1}))\Delta y(k+1) + b(z^{-1})\Delta u(k) \quad (4)$$

$$e(k+1) = e(k) + \Delta r(k+1) - \Delta y(k+1). \quad (5)$$

For the networked control of system (1), feedback packets and control packets are transmitted over the networks between the controller and the plant by using the standard Ethernet communication over the UDP/IP protocol. Thus, there usually exist network-induced delay, packet disorder, and packet dropout in the feedback and forward channels. To eliminate the adverse effects of these communication constraints, an INPC scheme is proposed as shown in Fig. 1. It consists of three parts: a data buffer (DB) in the sensor, a control prediction generator (CPG) in the controller, and a network

delay compensator (NDC) in the actuator. The function and design of DB, CPG, and NDC will be addressed in the following subsections.

For the design of INPC scheme, the following assumptions are first made:

Assumption 1: The sensor and actuator are time-driven and synchronous, whereas the controller is event-driven.

Assumption 2: The random RTT delay τ_k has an upper bound $\bar{\tau}$, i.e., $\tau_k \leq \bar{\tau}$.

Assumption 3: The packet transmitted over networks is with a timestamp.

Remark 1: Generally speaking, the packets lost in the networked loop and the out-of-order packets in the actuator are useless for the real-time control. In other words, we just need to focus on the packets *successfully* arriving at the actuator *in order*. Therefore, the RTT delay τ_k in Assumption 3 is redefined in this paper, which denotes the total time delay of the packet successfully arriving at the actuator in order. It can be obtained in the actuator by subtracting the timestamp of the latest packet available in the actuator from the current time of the actuator at each sampling instant. As a result, the RTT delay can represent the joint effects of the network-induced delay, packet disorder, and packet dropout in the feedback and forward channels.

A. Design of DB

The DB is designed to buffer the following measurement outputs, control inputs, and reference signals of the plant:

$$D_{k_s} = [Y(k_s)^T \ U(k_s-1)^T \ R(k_s-n_e)^T]^T \quad (6)$$

where

$$Y(k_s) = [y(k_s) \ y(k_s-1) \ \dots \ y(k_s-m)]^T$$

$$U(k_s-1) = [u(k_s-1) \ u(k_s-2) \ \dots \ u(k_s-n_b-1)]^T$$

$$R(k_s-n_e) = [r(k_s-n_e) \ r(k_s-n_e+1) \ \dots \ r(k_s+\bar{\tau})]^T$$

k_s is the timestamp, and $m = \max(n_a, n_e)$. At each sampling instant, the sensor packs the above data and the timestamp k_s into one packet and sends it to the controller.

B. Design of CPG

To simplify the subsequent formulation, define the following operations in this paper.

$$\Delta y(k_s + i|k_s) = \Delta y(k_s + i), \quad \text{if } i \leq 0 \quad (7)$$

$$e(k_s + i|k_s) = e(k_s + i), \quad \text{if } i \leq 0 \quad (8)$$

$$\Delta u(k_s + i|k_s) = \Delta u(k_s + i), \quad \text{if } i < 0 \quad (9)$$

where i is an integer, and $\Delta y(k_s + i|k_s)$, $e(k_s + i|k_s)$, and $\Delta u(k_s + i|k_s)$ are the i th-step-ahead predicted values of $\Delta y(k_s)$, $e(k_s)$, and $\Delta u(k_s)$ based on the data up to time k_s , respectively.

The CPG is designed to generate a sequence of control predictions based on the model of the plant. In many practical applications, however, it is usually difficult to obtain an accurate model of a dynamic plant. Without loss of generality,

suppose that the following polynomials $\hat{a}(z^{-1})$ and $\hat{b}(z^{-1})$ are available for the system polynomials $a(z^{-1})$ and $b(z^{-1})$ in (1):

$$\begin{cases} \hat{a}(z^{-1}) = 1 + \hat{a}_1 z^{-1} + \dots + \hat{a}_{n_a} z^{-n_a} \\ \hat{b}(z^{-1}) = \hat{b}_0 + \hat{b}_1 z^{-1} + \dots + \hat{b}_{n_b} z^{-n_b}. \end{cases} \quad (10)$$

When the feedback data in (6) arrive at the controller, the control law in (2) is used to calculate the control increment

$$\Delta u(k_s | k_s) = KE(k_s). \quad (11)$$

With the model parameters in (10), the predictions of the output increment, output tracking error, and control increment up to time $k_s + \bar{\tau}$ can be obtained by the iteration of (4), (5), and (11):

$$\begin{aligned} \Delta y(k_s + i | k_s) &= (1 - \hat{a}(z^{-1})) \Delta y(k_s + i | k_s) \\ &\quad + \hat{b}(z^{-1}) \Delta u(k_s + i - 1 | k_s) \end{aligned} \quad (12)$$

$$e(k_s + i | k_s) = e(k_s + i - 1 | k_s) - \Delta y(k_s + i | k_s) + \Delta r(k_s + i) \quad (13)$$

$$\Delta u(k_s + i | k_s) = KE(k_s + i | k_s) \quad (14)$$

for $i = 1, 2, \dots, \bar{\tau}$, where $E(k_s + i | k_s) = [e(k_s + i | k_s) \ e(k_s + i - 1 | k_s) \ \dots \ e(k_s + i - n_e | k_s)]^T$. Then from (11) and (14), we have

$$u(k_s + i | k_s) = u(k_s + i - 1 | k_s) + \Delta u(k_s + i | k_s) \quad (15)$$

for $i = 0, 1, 2, \dots, \bar{\tau}$, which yields the following control prediction sequence:

$$U_{k_s} = [u(k_s | k_s) \ u(k_s + 1 | k_s) \ \dots \ u(k_s + \bar{\tau} | k_s)]^T. \quad (16)$$

It is lumped into one packet together with the timestamp k_s and transmitted to the actuator.

C. Design of NDC

Due to the random network-induced delay, packet disorder, and packet dropout between the controller and the plant, it probably happens that one, more than one, or no control packets arrive at the actuator during one sampling interval. Suppose that at time k , the latest control prediction sequence available in the actuator is

$$U_{k_s^*} = [u(k_s^* | k_s^*) \ u(k_s^* + 1 | k_s^*) \ \dots \ u(k_s^* + \bar{\tau} | k_s^*)]^T \quad (17)$$

where k_s^* is its timestamp. Thus, the real-time RTT delay can be obtained:

$$\tau_k = k - k_s^*. \quad (18)$$

In order to compensate for the RTT delay, the NDC selects the following control signal for system (1):

$$u(k) = U_{k_s^*}(\tau_k) = u(k_s^* + \tau_k | k_s^*) = u(k | k - \tau_k) \quad (19)$$

which is equivalent to the case that the following control increment is applied to system (4):

$$\Delta u(k) = \Delta u(k | k - \tau_k). \quad (20)$$

III. ANALYSIS OF PERFORMANCE AND STABILITY

This section is concerned with the analysis of the output tracking performance and closed-loop stability of the resulting INPC system for two cases: plant-model match case and plant-model mismatch case.

A. Plant-Model Match Case

In this case, i.e., $\hat{a}(z^{-1}) = a(z^{-1})$ and $\hat{b}(z^{-1}) = b(z^{-1})$, the following theorem gives the output tracking performance of the INPC system.

Theorem 1: For time-varying reference signal $r(k) = y_0$ for $k < k_0$, where $k_0 \geq \bar{\tau}$ and y_0 is a steady-state value of $y(k)$, when $\hat{a}(z^{-1}) = a(z^{-1})$ and $\hat{b}(z^{-1}) = b(z^{-1})$, the INPC system can achieve the same output tracking performance as the corresponding local control system (LCS)¹.

Proof: To begin with, consider the case $k_0 = \bar{\tau}$. For the INPC system, Eqs. (12), (13), and (14) can be rewritten in the vector form as

$$\Delta Y(k + i + 1 | k) = \mathbf{A} \Delta Y(k + i | k) + \mathbf{B} \Delta U(k + i | k) \quad (21)$$

$$\begin{aligned} E(k + i + 1 | k) &= \mathbf{C}_e E(k + i | k) + \mathbf{A}_e \Delta Y(k + i | k) \\ &\quad + \mathbf{B}_e \Delta U(k + i | k) + D_e \Delta r(k + i + 1) \end{aligned} \quad (22)$$

$$\Delta U(k + i | k) = \mathbf{C} \Delta U(k + i - 1 | k) + \mathbf{D} E(k + i | k) \quad (23)$$

for $i = 0, 1, 2, \dots, \bar{\tau}$, where $E(k + i | k)$ is defined in (14), and

$$\begin{aligned} \Delta Y(k + i | k) &= [\Delta y(k + i | k) \ \Delta y(k + i - 1 | k) \ \dots \\ &\quad \Delta y(k + i - n_a + 1 | k)]^T \end{aligned}$$

$$\begin{aligned} \Delta U(k + i | k) &= [\Delta u(k + i | k) \ \Delta u(k + i - 1 | k) \ \dots \\ &\quad \Delta u(k + i - n_b | k)]^T \end{aligned}$$

$$\mathbf{A} = \begin{bmatrix} -a_1 & -a_2 & \dots & -a_{n_a} \\ & \mathbf{I}_{n_a-1} & & \mathbf{0}_{(n_a-1) \times 1} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} b_0 & b_1 & \dots & b_{n_b} \\ \mathbf{0}_{(n_a-1) \times (n_b+1)} \end{bmatrix}, \quad \mathbf{A}_e = \begin{bmatrix} a_1 & a_2 & \dots & a_{n_a} \\ & \mathbf{0}_{n_e \times n_a} \end{bmatrix}$$

$$\mathbf{B}_e = \begin{bmatrix} -b_0 & -b_1 & \dots & -b_{n_b} \\ & \mathbf{0}_{n_e \times (n_b+1)} \end{bmatrix}$$

$$\mathbf{C}_e = \begin{bmatrix} 1 & \mathbf{0}_{1 \times n_e} \\ \mathbf{I}_{n_e} & \mathbf{0}_{n_e \times 1} \end{bmatrix}, \quad D_e = \begin{bmatrix} 1 \\ \mathbf{0}_{n_e \times 1} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{0}_{1 \times (n_b+1)} \\ \mathbf{I}_{n_b} & \mathbf{0}_{n_b \times 1} \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} K \\ \mathbf{0}_{n_b \times (n_e+1)} \end{bmatrix}.$$

Combining (21), (22), and (23) gives

$$\begin{aligned} X(k + i + 1 | k) &= \mathbf{\Lambda} X(k + i | k) + \Gamma \Delta r(k + i + 1) \\ &= \mathbf{\Lambda}^{i+1} X(k) + \sum_{j=0}^i \mathbf{\Lambda}^j \Gamma \Delta r(k + i + 1 - j) \end{aligned} \quad (24)$$

for $i = 0, 1, 2, \dots, \bar{\tau}$, where

$$X(k + i | k) = [E(k + i | k)^T \ \Delta Y(k + i | k)^T \ \Delta U(k + i - 1 | k)^T]^T$$

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{C}_e + \mathbf{B}_e \mathbf{D} & \mathbf{A}_e & \mathbf{B}_e \mathbf{C} \\ \mathbf{B} \mathbf{D} & \mathbf{A} & \mathbf{B} \mathbf{C} \\ \mathbf{D} & \mathbf{0}_{(n_b+1) \times n_a} & \mathbf{C} \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} 1 \\ \mathbf{0}_{(\bar{n}-1) \times 1} \end{bmatrix}, \quad \bar{n} = n_a + n_b + n_e + 2$$

$$X(k) = X(k | k) = [E(k)^T \ \Delta Y(k)^T \ \Delta U(k - 1)^T]^T$$

¹The INPC system denotes (1) with (20), and the LCS is (1) with (2).

$$\Delta Y(k) = [\Delta y(k) \Delta y(k-1) \cdots \Delta y(k-n_a+1)]^T$$

$$\Delta U(k-1) = [\Delta u(k-1) \Delta u(k-2) \cdots \Delta u(k-n_b-1)]^T$$

and $E(k)$ is defined in (2).

Without loss of generality, suppose that $y(k) = y_0$ and $u(k-1) = u_0$ for $k \leq \bar{\tau}$, where $u_0 \in \mathbb{R}$ is a steady-state value of $u(k)$. With the reference signal $r(k) = y_0$ for $k < \bar{\tau}$, in view of (24) and (20), we can obtain in turn for $k = 0, 1, 2, \dots, \bar{\tau} - 1$:

$$X(k) = X(0) = 0_{\bar{n} \times 1} \quad (25)$$

$$X(k+i+1|k) = \begin{cases} 0_{\bar{n} \times 1}, & \text{if } k+i+1 < \bar{\tau} \\ \Lambda_s^{k+i+1-\bar{\tau}}, & \text{if } k+i+1 \geq \bar{\tau} \end{cases} \quad (26)$$

$$\Delta u(k+i|k) = I_u X(k+i+1|k) \quad (27)$$

$$\begin{aligned} \Delta U_k &= [\Delta u(k|k) \Delta u(k+1|k) \cdots \Delta u(k+\bar{\tau}|k)]^T \\ &= [0_{1 \times (\bar{\tau}-k)} \quad I_u \Lambda_s^1 \quad I_u \Lambda_s^2 \cdots I_u \Lambda_s^{k+1}]^T \end{aligned} \quad (28)$$

$$\Delta u(k) = \Delta U_{k-\tau_k}(\tau_k) = 0 \quad (29)$$

where $i = 0, 1, 2, \dots, \bar{\tau}$, $I_u = [0_{1 \times (n_e+n_a+1)} \quad 1 \quad 0_{1 \times n_b}]$, and

$$\Lambda_s^l = \sum_{j=0}^l \mathbf{\Lambda}^j \Gamma \Delta r(\bar{\tau} + l - j)$$

where $l \geq 0$ is an integer. It can be seen from (29) that at each time instant $k = 0, 1, 2, \dots, \bar{\tau} - 1$, no matter whether or not the actuator receives any control increment sequences in (28), the control increment applied to system (4) is always $\Delta u(k) = 0$.

At time $k = \bar{\tau}$, we have

$$X(\bar{\tau}) = \Gamma \Delta r(\bar{\tau}) = \Lambda_s^0 \quad (30)$$

$$\begin{aligned} X(\bar{\tau} + i + 1|\bar{\tau}) &= \mathbf{\Lambda}^{i+1} X(\bar{\tau}) + \sum_{j=0}^i \mathbf{\Lambda}^j \Gamma \Delta r(\bar{\tau} + i + 1 - j) \\ &= \sum_{j=0}^{i+1} \mathbf{\Lambda}^j \Gamma \Delta r(\bar{\tau} + i + 1 - j) = \Lambda_s^{i+1} \end{aligned} \quad (31)$$

$$\Delta U_{\bar{\tau}} = [I_u \Lambda_s^1 \quad I_u \Lambda_s^2 \cdots I_u \Lambda_s^{\bar{\tau}+1}]^T \quad (32)$$

$$\Delta u(\bar{\tau}) = \Delta U_{\bar{\tau}-\tau_{\bar{\tau}}}(\tau_{\bar{\tau}}) = I_u \Lambda_s^1. \quad (33)$$

Due to the upper bound $\bar{\tau}$ of RTT delay, at time $k = \bar{\tau}$, at least one of control increment sequences in (28) and (32) is available in the actuator. Eq. (33) indicates that, no matter which one is adopted, i.e., no matter what value $\tau_{\bar{\tau}} \in [0, \bar{\tau}]$ takes, the control increment applied to the system (4) is always $\Delta u(\bar{\tau}) = I_u \Lambda_s^1$. Furthermore, we have

$$E(\bar{\tau}) = \mathbf{I}_E X(\bar{\tau}) = \mathbf{I}_E \Lambda_s^0 = E(\bar{\tau}|\bar{\tau} - \tau_{\bar{\tau}}) \quad (34)$$

$$\Delta Y(\bar{\tau}) = \mathbf{I}_Y X(\bar{\tau}) = \mathbf{I}_Y \Lambda_s^0 = \Delta Y(\bar{\tau}|\bar{\tau} - \tau_{\bar{\tau}}) \quad (35)$$

$$\Delta U(\bar{\tau}) = \Delta U(\bar{\tau}|\bar{\tau} - \tau_{\bar{\tau}}) = \mathbf{I}_U \Lambda_s^1 \quad (36)$$

where

$$\begin{cases} \mathbf{I}_E = [I_{n_e+1} \quad \mathbf{0}_{(n_e+1) \times (n_a+n_b+1)}] \\ \mathbf{I}_Y = [\mathbf{0}_{n_a \times (n_e+1)} \quad I_{n_a} \quad \mathbf{0}_{n_a \times (n_b+1)}] \\ \mathbf{I}_U = [\mathbf{0}_{(n_b+1) \times (n_e+n_a+1)} \quad I_{n_b+1}]. \end{cases}$$

For the plant in (1), it can be obtained from (4) and (5) that

$$\Delta Y(k+1) = \mathbf{A} \Delta Y(k) + \mathbf{B} \Delta U(k) \quad (37)$$

$$E(k+1) = \mathbf{C}_e E(k) + \mathbf{A}_e \Delta Y(k) + \mathbf{B}_e \Delta U(k) + \mathbf{D}_e \Delta r(k+1). \quad (38)$$

Using (34)-(36), it can be obtained from the comparison between (21), (22) and (37), (38) that

$$E(\bar{\tau} + 1) = E(\bar{\tau} + 1|\bar{\tau} - \tau_{\bar{\tau}}) \quad (39)$$

$$\Delta Y(\bar{\tau} + 1) = \Delta Y(\bar{\tau} + 1|\bar{\tau} - \tau_{\bar{\tau}}). \quad (40)$$

Then, with (36), we have

$$X(\bar{\tau} + 1) = X(\bar{\tau} + 1|\bar{\tau} - \tau_{\bar{\tau}}) = \Lambda_s^1. \quad (41)$$

Similarly, it can be deduced that

$$X(k+1) = X(k+1|k - \tau_k) = \Lambda_s^{k+1-\bar{\tau}} \quad (42)$$

for $k \geq \bar{\tau}$.

For the LCS, the closed-loop system can be obtained from (37), (38), and (2) as follows:

$$X_L(k+1) = \mathbf{\Lambda} X_L(k) + \Gamma \Delta r(k+1) \quad (43)$$

where $X_L(k)$ has the same form as $X(k)$ defined in (24). With $y(k) = y_0$ and $u(k-1) = u_0$ for $k \leq \bar{\tau}$, as well as the reference signal $r(k) = y_0$ for $k < \bar{\tau}$, from (43), we have

$$X_L(k) = 0_{\bar{n} \times 1} \quad (44)$$

for $k < \bar{\tau}$, and

$$X_L(k) = \mathbf{\Lambda}^{k-\bar{\tau}+1} X_L(\bar{\tau}-1) + \sum_{j=0}^{k-\bar{\tau}} \mathbf{\Lambda}^j \Gamma \Delta r(k-j) = \Lambda_s^{k-\bar{\tau}} \quad (45)$$

for $k \geq \bar{\tau}$.

Therefore, it can be obtained from (25), (30), (42) and (44), (45) that

$$X(k) = X_L(k) = \begin{cases} 0_{\bar{n} \times 1}, & \text{if } k < \bar{\tau} \\ \Lambda_s^{k-\bar{\tau}}, & \text{if } k \geq \bar{\tau}. \end{cases} \quad (46)$$

By using the similar procedure, we can obtain for the case $k_0 \geq \bar{\tau}$:

$$X(k) = X_L(k) = \begin{cases} 0_{\bar{n} \times 1}, & \text{if } k < k_0 \\ \Lambda_{s_0}^{k-k_0}, & \text{if } k \geq k_0 \end{cases} \quad (47)$$

where

$$\Lambda_{s_0}^l = \sum_{j=0}^l \mathbf{\Lambda}^j \Gamma \Delta r(k_0 + l - j).$$

With $e(k) = I_e X(k)$, $I_e = \Gamma^T$, it can be obtained from (47) that the output tracking error of the INPC system is equal to that of the LCS at all time k . The proof is completed. ■

Remark 2: It is worth noting that in Theorem 1, the input constraints of the controlled plant are not considered. Suppose that the control input is bounded by

$$u_{\min} \leq u(k) \leq u_{\max} \quad (48)$$

where u_{\min} and u_{\max} are the lower and upper bounds of $u(k)$. To cope with the input constraints in (48), with (11) and (13)-(15) unchanged, the CPG needs to be redesigned as

$$u_c(k_s + i|k_s) = \begin{cases} u_{\max}, & \text{if } u(k_s + i|k_s) > u_{\max} \\ u_{\min}, & \text{if } u(k_s + i|k_s) < u_{\min} \\ u(k_s + i|k_s), & \text{otherwise} \end{cases} \quad (49)$$

$$\Delta u_c(k_s + i|k_s) = u_c(k_s + i|k_s) - u_c(k_s + i - 1|k_s) \quad (50)$$

for $i = 0, 1, 2, \dots, \bar{\tau}$, and

$$\Delta y(k_s + i|k_s) = (1 - \hat{a}(z^{-1}))\Delta y(k_s + i|k_s) + \hat{b}(z^{-1})\Delta u_c(k_s + i - 1|k_s) \quad (51)$$

for $i = 1, 2, \dots, \bar{\tau}$. Then the following control prediction sequence

$$U_{k_s} = [u_c(k_s|k_s) \ u_c(k_s + 1|k_s) \ \dots \ u_c(k_s + \bar{\tau}|k_s)]^T \quad (52)$$

is sent to the actuator together with the timestamp k_s . Thus, from the above derivation procedure of Theorem 1, it can be deduced that with the input constraints in (48), Theorem 1 still holds for the plant-model match case, which will be confirmed by the simulation results in the next section.

It can also be seen from (47) that the INPC system has the same closed-loop stability as that of the LCS. Therefore, from (43), we readily obtain the following corollary.

Corollary 1: When $\hat{a}(z^{-1}) = a(z^{-1})$ and $\hat{b}(z^{-1}) = b(z^{-1})$, the closed-loop INPC system is globally asymptotically stable if and only if the eigenvalues of matrix \mathbf{A} are within the unit circle.

Next, we further present the result on the output tracking performance of INPC system for the step reference signal.

Theorem 2: For the step reference signal

$$r(k) = \begin{cases} y_0, & \text{if } k < k_0 \\ y_0 + \bar{r}, & \text{if } k \geq k_0 \end{cases} \quad (53)$$

where $k_0 \geq \bar{\tau}$ and $\bar{r} \in \mathbb{R}$ is a constant, when $\hat{a}(z^{-1}) = a(z^{-1})$ and $\hat{b}(z^{-1}) = b(z^{-1})$, the INPC system can achieve a zero steady-state output tracking error if the eigenvalues of matrix \mathbf{A} are within the unit circle.

Proof: With the step reference signal in (53), we have

$$\Delta r(k) = \begin{cases} \bar{r}, & \text{if } k = k_0 \\ 0, & \text{otherwise} \end{cases} \quad (54)$$

Since the eigenvalues of matrix \mathbf{A} are within the unit circle, the steady-state output tracking error is obtained from (47) as

$$\begin{aligned} \lim_{k \rightarrow \infty} e(k) &= \lim_{k \rightarrow \infty} I_e \Lambda_{s0}^{k-k_0} \\ &= \lim_{k \rightarrow \infty} I_e \sum_{j=0}^{k-k_0} \mathbf{A}^j \Gamma \Delta r(k-j) \\ &= \lim_{k \rightarrow \infty} I_e \Lambda^{k-k_0} \Gamma \bar{r} = 0 \end{aligned} \quad (55)$$

The proof is completed. \blacksquare

B. Plant-Model Mismatch Case

In this case, $\hat{a}(z^{-1}) \neq a(z^{-1})$ or $\hat{b}(z^{-1}) \neq b(z^{-1})$. It can be obtained for the INPC system from (12), (13), and (14) that

$$\begin{aligned} X(k+1|k-\tau_k) &= \hat{\mathbf{A}}X(k|k-\tau_k) + \Gamma \Delta r(k+1) \\ &= \hat{\mathbf{A}}^{\tau_k+1} X(k-\tau_k) + \sum_{j=0}^{\tau_k} \hat{\mathbf{A}}^j \Gamma \Delta r(k+1-j) \end{aligned} \quad (56)$$

where

$$\begin{aligned} \hat{\mathbf{A}} &= \begin{bmatrix} \mathbf{C}_e + \hat{\mathbf{B}}_e \mathbf{D} & \hat{\mathbf{A}}_e & \hat{\mathbf{B}}_e \mathbf{C} \\ \hat{\mathbf{B}} \mathbf{D} & \hat{\mathbf{A}} & \hat{\mathbf{B}} \mathbf{C} \\ \mathbf{D} & \mathbf{0}_{(n_b+1) \times n_a} & \mathbf{C} \end{bmatrix} \\ \hat{\mathbf{A}} &= \begin{bmatrix} -\hat{a}_1 & -\hat{a}_2 & \dots & -\hat{a}_{n_a} \\ \mathbf{I}_{n_a-1} & \mathbf{0}_{(n_a-1) \times 1} & & \end{bmatrix} \\ \hat{\mathbf{B}} &= \begin{bmatrix} \hat{b}_0 & \hat{b}_1 & \dots & \hat{b}_{n_b} \\ \mathbf{0}_{(n_a-1) \times (n_b+1)} \end{bmatrix}, \quad \hat{\mathbf{A}}_e = \begin{bmatrix} \hat{a}_1 & \hat{a}_2 & \dots & \hat{a}_{n_a} \\ \mathbf{0}_{n_e \times n_a} \end{bmatrix} \\ \hat{\mathbf{B}}_e &= \begin{bmatrix} -\hat{b}_0 & -\hat{b}_1 & \dots & -\hat{b}_{n_b} \\ \mathbf{0}_{n_e \times (n_b+1)} \end{bmatrix}. \end{aligned}$$

From (20), we have

$$\begin{aligned} \Delta u(k) &= I_u X(k+1|k-\tau_k) \\ &= F_{\tau_k} X(k-\tau_k) + \sum_{j=0}^{\tau_k} h_j \Delta r(k+1-j) \end{aligned} \quad (57)$$

where $F_{\tau_k} = I_u \hat{\mathbf{A}}^{\tau_k+1} = [f_{\tau_k,1} \ f_{\tau_k,2} \ \dots \ f_{\tau_k,\bar{n}}]$, and $h_j = I_u \hat{\mathbf{A}}^j \Gamma$, $j = 0, 1, 2, \dots, \tau_k$. Eq. (57) can be rewritten as

$$\begin{aligned} \Delta \tilde{U}(k) &= \tilde{\mathbf{C}}_{\tau_k} \Delta \tilde{U}(k-1) + \tilde{\mathbf{D}}_{\tau_k} \tilde{\mathbf{E}}(k) + \tilde{\mathbf{F}}_{\tau_k} \Delta \tilde{Y}(k) \\ &\quad + \tilde{\mathbf{H}}_{\tau_k} \Delta \tilde{R}(k+1) \end{aligned} \quad (58)$$

where

$$\begin{aligned} \tilde{\mathbf{E}}(k) &= [e(k) \ e(k-1) \ \dots \ e(k-\bar{\tau}-n_e)]^T \\ \Delta \tilde{Y}(k) &= [\Delta y(k) \ \Delta y(k-1) \ \dots \ \Delta y(k-\bar{\tau}-n_a+1)]^T \\ \Delta \tilde{U}(k) &= [\Delta u(k) \ \Delta u(k-1) \ \dots \ \Delta u(k-\bar{\tau}-n_b)]^T \\ \Delta \tilde{R}(k+1) &= [\Delta r(k+1) \ \Delta r(k) \ \dots \ \Delta r(k+1-\bar{\tau})]^T \\ \tilde{\mathbf{C}}_{\tau_k} &= \begin{bmatrix} \mathbf{0}_{1 \times \tau_k} & f_{\tau_k, n_e+n_a+2} & \dots & f_{\tau_k, \bar{n}} & \mathbf{0}_{1 \times (\bar{\tau}-\tau_k)} \\ & \mathbf{I}_{n_b+\bar{\tau}} & & & \mathbf{0}_{(n_b+\bar{\tau}) \times 1} \end{bmatrix} \\ \tilde{\mathbf{D}}_{\tau_k} &= \begin{bmatrix} \mathbf{0}_{1 \times \tau_k} & f_{\tau_k,1} & \dots & f_{\tau_k, n_e+1} & \mathbf{0}_{1 \times (\bar{\tau}-\tau_k)} \\ & \mathbf{0}_{(n_b+\bar{\tau}) \times (n_e+\bar{\tau}+1)} & & & \end{bmatrix} \\ \tilde{\mathbf{F}}_{\tau_k} &= \begin{bmatrix} \mathbf{0}_{1 \times \tau_k} & f_{\tau_k, n_e+2} & \dots & f_{\tau_k, n_e+n_a+1} & \mathbf{0}_{1 \times (\bar{\tau}-\tau_k)} \\ & \mathbf{0}_{(n_b+\bar{\tau}) \times (n_a+\bar{\tau})} & & & \end{bmatrix} \\ \tilde{\mathbf{H}}_{\tau_k} &= \begin{bmatrix} h_0 & h_1 & \dots & h_{\tau_k} & \mathbf{0}_{1 \times (\bar{\tau}-\tau_k)} \\ & \mathbf{0}_{(n_b+\bar{\tau}) \times (\bar{\tau}+1)} & & & \end{bmatrix}. \end{aligned}$$

Eqs. (4) and (5) can be rewritten as

$$\Delta \tilde{Y}(k+1) = \tilde{\mathbf{A}} \Delta \tilde{Y}(k) + \tilde{\mathbf{B}} \Delta \tilde{U}(k) \quad (59)$$

$$\tilde{\mathbf{E}}(k+1) = \tilde{\mathbf{C}}_e \tilde{\mathbf{E}}(k) + \tilde{\mathbf{A}}_e \Delta \tilde{Y}(k) + \tilde{\mathbf{B}}_e \Delta \tilde{U}(k) + \tilde{\mathbf{D}}_e \Delta \tilde{R}(k+1) \quad (60)$$

where

$$\tilde{\mathbf{A}} = \begin{bmatrix} -a_1 & -a_2 & \dots & -a_{n_a} & \mathbf{0}_{1 \times \bar{\tau}} \\ \mathbf{I}_{n_a+\bar{\tau}-1} & & & & \mathbf{0}_{(n_a+\bar{\tau}-1) \times 1} \end{bmatrix}$$

$$\begin{aligned}\tilde{\mathbf{B}} &= \begin{bmatrix} b_0 & b_1 & \cdots & b_{n_b} & \mathbf{0}_{1 \times \bar{\tau}} \\ \mathbf{0}_{(n_a + \bar{\tau} - 1) \times (n_b + \bar{\tau} + 1)} & & & & \end{bmatrix} \\ \tilde{\mathbf{A}}_e &= \begin{bmatrix} a_1 & a_2 & \cdots & a_{n_a} & \mathbf{0}_{1 \times \bar{\tau}} \\ \mathbf{0}_{(n_e + \bar{\tau}) \times (n_a + \bar{\tau})} & & & & \end{bmatrix} \\ \tilde{\mathbf{B}}_e &= \begin{bmatrix} -b_0 & -b_1 & \cdots & -b_{n_b} & \mathbf{0}_{1 \times \bar{\tau}} \\ \mathbf{0}_{(n_e + \bar{\tau}) \times (n_b + \bar{\tau} + 1)} & & & & \end{bmatrix} \\ \tilde{\mathbf{C}}_e &= \begin{bmatrix} 1 & \mathbf{0}_{1 \times (n_e + \bar{\tau})} \\ \mathbf{I}_{n_e + \bar{\tau}} & \mathbf{0}_{(n_e + \bar{\tau}) \times 1} \end{bmatrix}, \quad \tilde{\mathbf{D}}_e = \begin{bmatrix} 1 & \mathbf{0}_{1 \times \bar{\tau}} \\ \mathbf{0}_{(n_e + \bar{\tau}) \times (\bar{\tau} + 1)} \end{bmatrix}.\end{aligned}$$

Combining (58), (59), and (60) yields the following closed-loop INPC system:

$$\tilde{X}(k+1) = \tilde{\Lambda}(\tau_k)\tilde{X}(k) + \tilde{\Gamma}(\tau_k)\Delta\tilde{R}(k+1) \quad (61)$$

where

$$\begin{aligned}\tilde{X}(k) &= [\tilde{E}(k)^T \quad \Delta\tilde{Y}(k)^T \quad \Delta\tilde{U}(k-1)^T]^T \\ \tilde{\Lambda}(\tau_k) &= \begin{bmatrix} \tilde{\mathbf{C}}_e + \tilde{\mathbf{B}}_e\tilde{\mathbf{D}}_{\tau_k} & \tilde{\mathbf{A}}_e + \tilde{\mathbf{B}}_e\tilde{\mathbf{F}}_{\tau_k} & \tilde{\mathbf{B}}_e\tilde{\mathbf{C}}_{\tau_k} \\ \tilde{\mathbf{B}}\tilde{\mathbf{D}}_{\tau_k} & \tilde{\mathbf{A}} + \tilde{\mathbf{B}}\tilde{\mathbf{F}}_{\tau_k} & \tilde{\mathbf{B}}\tilde{\mathbf{C}}_{\tau_k} \\ \tilde{\mathbf{D}}_{\tau_k} & \tilde{\mathbf{F}}_{\tau_k} & \tilde{\mathbf{C}}_{\tau_k} \end{bmatrix} \\ \tilde{\Gamma}(\tau_k) &= \begin{bmatrix} \tilde{\mathbf{D}}_e + \tilde{\mathbf{B}}_e\tilde{\mathbf{H}}_{\tau_k} \\ \tilde{\mathbf{B}}\tilde{\mathbf{H}}_{\tau_k} \\ \tilde{\mathbf{H}}_{\tau_k} \end{bmatrix}.\end{aligned}$$

Since the RTT delay τ_k randomly takes value in the finite set $\ell = \{0, 1, 2, \dots, \bar{\tau}\}$, the system (61) is a linear switched system. Therefore, the following stability theorem can be obtained for the closed-loop system (61).

Theorem 3: When $\hat{a}(z^{-1}) \neq a(z^{-1})$ or $\hat{b}(z^{-1}) \neq b(z^{-1})$, the closed-loop INPC system (61) is globally asymptotically stable if there exist $\bar{\tau} + 1$ positive definite matrices $\mathbf{P}(\tau_k)$ satisfying

$$\tilde{\Lambda}^T(\tau_k)\mathbf{P}(\tau_{k+1})\tilde{\Lambda}(\tau_k) - \mathbf{P}(\tau_k) < \mathbf{0} \quad (62)$$

for all $(\tau_k, \tau_{k+1}) \in \ell \times \ell$.

Proof: The proof can refer to [25] and is omitted here. ■

Remark 3: Clearly, Theorem 3 is also suitable for the plant-model match case of the INPC system, but compared with Corollary 1, it has considerable conservativeness, which will be confirmed by the simulation example in Section IV. However, only sufficient stability conditions like Theorem 3 were given in [21] and [23]-[25] for the plant-model match case of the corresponding NPC system.

Based on Theorem 3, the following theorem can then be obtained for the performance of the INPC system with plant-model mismatch for the step reference signal.

Theorem 4: For the step reference signal in (53), when $\hat{a}(z^{-1}) \neq a(z^{-1})$ or $\hat{b}(z^{-1}) \neq b(z^{-1})$, the INPC system can achieve a zero steady-state output tracking error if the closed-loop INPC system (61) is globally asymptotically stable.

Proof: With the step reference signal in (53), we have

$$\tilde{X}(k) = \mathbf{0}_{(\bar{n}+3\bar{\tau}) \times 1} \quad (63)$$

for $k < k_0$, and

$$\Delta\tilde{R}(k) = \begin{cases} \tilde{I}_k, & \text{if } k_0 \leq k \leq k_0 + \bar{\tau} \\ \mathbf{0}_{(\bar{\tau}+1) \times 1}, & \text{otherwise} \end{cases} \quad (64)$$

where $\tilde{I}_k = [0_{1 \times (k-k_0)} \quad 1 \quad 0_{1 \times (\bar{\tau}-k+k_0)}]^T$ for $k_0 \leq k \leq k_0 + \bar{\tau}$. Then, it learns from (61) that

$$\begin{aligned}\tilde{X}(k) &= \prod_{i=1}^{k-k_0+1} \tilde{\Lambda}(\tau_{k-i})\tilde{X}(k_0-1) + \tilde{\Gamma}(\tau_{k-1})\Delta\tilde{R}(k) \\ &\quad + \sum_{i=1}^{k-k_0} \prod_{j=1}^i \tilde{\Lambda}(\tau_{k-j})\tilde{\Gamma}(\tau_{k-i-1})\Delta\tilde{R}(k-i) \quad (65) \\ &= \sum_{i=i_0}^{k-k_0} \prod_{j=1}^i \tilde{\Lambda}(\tau_{k-j})\tilde{\Gamma}(\tau_{k-i-1})\tilde{I}_{k-i}\end{aligned}$$

for $k > k_0 + \bar{\tau}$, where $i_0 = k - k_0 - \bar{\tau}$.

Since the closed-loop system (61) is globally asymptotically stable, it can be obtained from Theorem 3 that, for all possible τ_k , the eigenvalues of matrix $\tilde{\Lambda}(\tau_k)$ are within the unit circle. Thus, the steady-state error is obtained from (65) as

$$\begin{aligned}\lim_{k \rightarrow \infty} e(k) &= \lim_{k \rightarrow \infty} \tilde{I}_e \tilde{X}(k) \\ &= \lim_{k \rightarrow \infty} \tilde{I}_e \sum_{i=i_0}^{k-k_0} \prod_{j=1}^i \tilde{\Lambda}(\tau_{k-j})\tilde{\Gamma}(\tau_{k-i-1})\tilde{I}_{k-i} \quad (66) \\ &= \mathbf{0}\end{aligned}$$

where $\tilde{I}_e = [1 \quad 0_{1 \times (\bar{n}+3\bar{\tau}-1)}]$. The proof is completed. ■

Remark 4: Compared with the NPC methods in [21]-[27], the proposed INPC method has two advantages. i) It is theoretically established in Theorem 1 that, for the plant-model match case, the INPC system can achieve the same output tracking performance as the corresponding LCS. Specifically, for both the plant-model match and mismatch cases, the INPC system can achieve a zero steady-state output tracking error for the step reference signal. However, in [21]-[26], the performance analysis was not involved, and the results obtained in [27] are conservative. To evaluate the output tracking performance of these NPC methods, only some qualitative judgments were made by simulation or experimental examples. Moreover, in [21]-[27], the NPC systems with the plant-model mismatch would cause a steady-state error for the step reference signal. ii) Compared with the sufficient conditions in [21]-[27], Corollary 1 gives a necessary and sufficient condition for the closed-loop stability for the plant-model match case, which is independent of RTT delays. Therefore, the design of the controller gain K in (11) and (14) can follow the design procedure of the corresponding LCS. Furthermore, the stability condition for the plant-model mismatch case is also given in Theorem 3.

IV. NUMERICAL SIMULATION

To illustrate the effectiveness of the proposed INPC method, a servo motor system (SMS) is considered, whose input and output are the control voltage (V) and the angle position ($^\circ$), respectively. For the sampling period 0.04s, the SMS can be described by

$$G(z^{-1}) = \frac{3.2099z^{-1} + 2.4072z^{-2} + 0.7906z^{-3}}{1 - 1.3195z^{-1} + 0.4242z^{-2} - 0.1046z^{-3}} \quad (67)$$

The controller gain K is designed to be

$$K = [0.041 \quad -0.040 \quad 0.002] \quad (68)$$

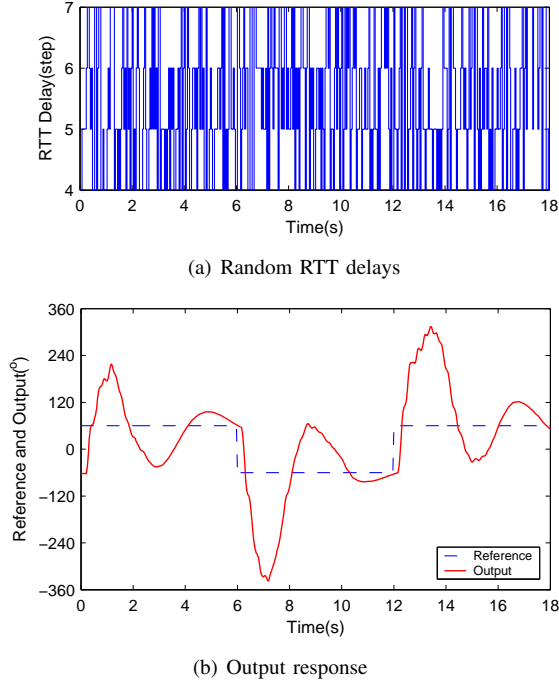


Fig. 2. Performance of NCS (simulation).

In the following simulations, the reference signal $r(k)$ is chosen as a square wave between -60° and 60° with the period 12s.

A. Networked Control System

The RTT delays are produced by the computer simulation, which randomly vary between 4 steps and 7 steps, as shown in Fig. 2(a). With these random RTT delays, the output response of NCS without network delay compensation is shown in Fig. 2(b), which indicates that the output tracking performance is very poor. If the RTT delays become larger, the NCS with network delay compensation will become unstable.

B. Incremental Networked Predictive Control System

1) *Plant-Model Match Case:* With the controller gain in (68), the non-zero eigenvalues of the matrix Λ in (24) are calculated to be $\{0.3225 \pm 0.2108i, 0.8839, 0.6768, -0.0178\}$. By using Corollary 1, the closed-loop INPC system is asymptotically stable. However, it fails to judge the stability of the closed-loop INPC system by Theorem 3, since the LMIs in (62) have no solution.

With the random RTT delays in Fig. 2(a), the simulation result of the INPC system is given in Fig. 3 (red solid line), which indicates that the INPC system is stable. Moreover, the output response is the same as that of the LCS (blue dotted line), which coincides with the result of Theorem 1. In addition, with the random RTT delays in Fig. 2(a) and the controller parameters in (68), the performance of the NPC method in [25] is also shown in Fig. 3 (black dash-dot line). It can be seen that, for the plant-model match case, the NPC method in [25] gives the same output tracking performance as the proposed INPC method in this paper.

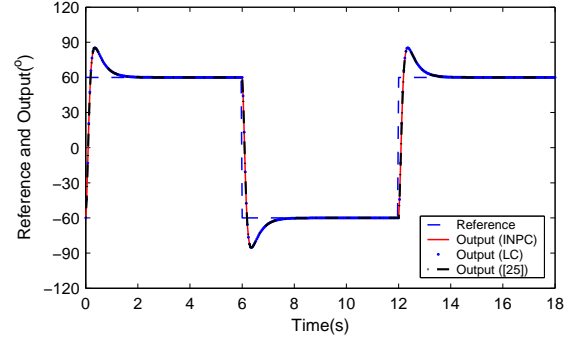


Fig. 3. Performance of INPC system (simulation).

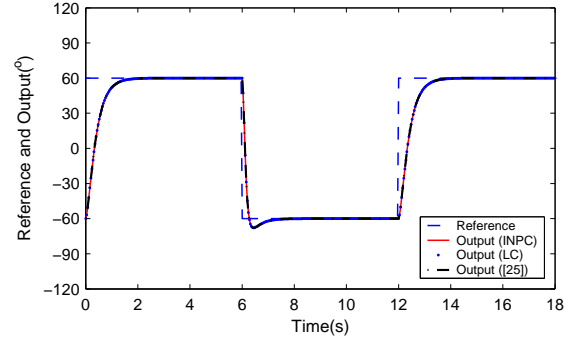


Fig. 4. Performance of INPC system with input constraints (simulation).

Next, to test the output tracking performance of the INPC method with input constraints, it is assumed that the control input of the SMS is bounded by $-4V \leq u(k) \leq 1V$. With the random RTT delays in Fig. 2(a), the simulation results are shown in Fig. 4. It can be seen that, for the plant-model match case, the output responses of the INPC system (red solid line) and the NPC system in [25] (black dash-dot line) are the same as that of the LCS (blue dotted line) with zero steady-state output tracking errors.

2) *Plant-Model Mismatch Case:* Suppose that, the following model polynomials are available for the SMS in (67):

$$\begin{cases} \hat{a}(z^{-1}) = 1 + 0.9a_1z^{-1} + 0.8a_2z^{-2} + 1.1a_3z^{-3} \\ \hat{b}(z^{-1}) = 0.7b(z^{-1}) \end{cases} \quad (69)$$

With the controller gain in (68), the positive definite matrices $P(i)$ for $i=4, 5, 6, 7$ are obtained by solving the LMIs in (62), of which the dimension is 30 so that their values are omitted here. Therefore, the closed-loop INPC system with the plant-model mismatch in (69) is stable by using Theorem 3.

With the random RTT delays in Fig. 2(a), the output response of the INPC system is shown in Fig. 5 (red line). It can be seen that the INPC system with the plant-model mismatch in (69) is still stable with zero steady-state errors, which coincides with the result of Theorem 4. On the other hand, for the same plant-model mismatch, the performance of the NPC method in [25] is shown in Fig. 5 (blue line). It can be seen that the plant-model mismatch in (69) leads to a significant steady-state output tracking error.

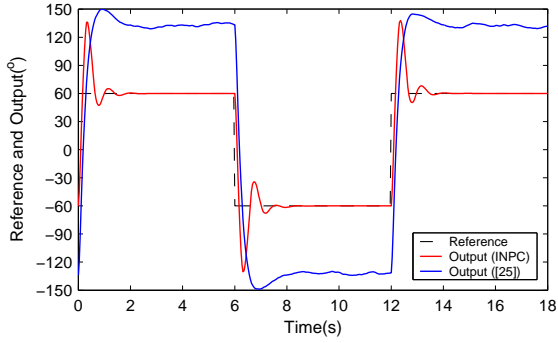


Fig. 5. Performance of INPC system with plant-model mismatch (simulation).

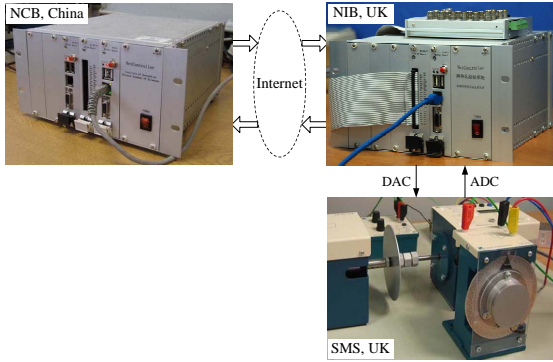


Fig. 6. Internet-based SMS.

V. PRACTICAL IMPLEMENTATION

A. Internet-Based Servo Motor System

To further verify the INPC method on practical systems, an Internet-based SMS test rig has been built as shown in Fig. 6, which consists of an SMS, a networked implementation board (NIB), a networked controller board (NCB), as well as the Internet between the NIB and the NCB. The SMS and the NIB are located in University of South Wales, Pontypridd, UK, which are directly connected by wires. The NCB is placed in Tsinghua University, Beijing, China, which is connected to the NIB through the Internet.

The SMS is mainly composed of a DC motor, a gear box, an angle position sensor, an amplifier and a power supply, which are magnetically assembled on a base plate. It is used for the control of angle position ($-120^\circ \sim 120^\circ$) in this paper, which is driven by the input voltage from $-10V$ to $10V$. The kernel chip of the NIB and the NCB is ATMEL AT91RM9200, which is a 32-bit RISC microcontroller for Ethernet-based embedded systems. There are 4M-byte NOR flash ROM, 64M-byte NAND flash ROM, and 32M-byte SDRAM memory in the two boards, which provide a powerful capability of data storage and efficient operation.

The NIB (193.63.131.219) has 12 analog-to-digital (A/D) channels and 4 digital-to-analog (D/A) channels, which is used as the interface between the SMS and the Internet. The NCB (166.111.72.24) is employed for the implementation of control strategies. For the real-time control, the standard UDP/IP protocol is adopted over the NIB-to-NCB Internet. The NCB

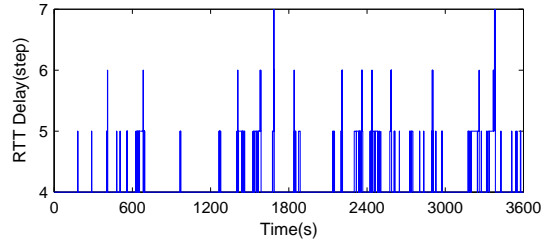


Fig. 7. RTT delays between NIB (UK) and NCB (China).

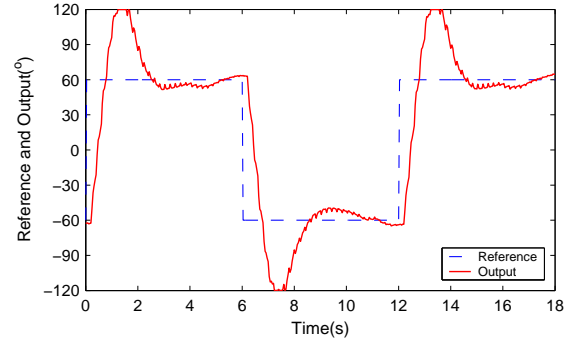


Fig. 8. Performance of NCS (experiment).

is set to be event-driven, and the NIB is time-driven. The sampling period for the networked control is 0.04s.

B. Practical Experiments

According to the actual experiment of one hour, the RTT delays between the NIB and the NCB are obtained and vary from 4 to 7 steps, which are shown in Fig. 7. In the following experiments, the reference signal $r(k)$ and the controller gain K are chosen to be the same as those in the simulations.

Firstly, the performance of the NCS without network delay compensation is tested. The output response of the NCS is shown in Fig. 8. It can be seen that, due to the random RTT delays in Fig. 7, the NCS gives a poor output tracking performance.

Secondly, with the random RTT delays in Fig. 7, the output response of the INPC system based on the model in (67) is shown in Fig. 9 (red line). It can be seen that the output tracking performance is well achieved with zero steady-state

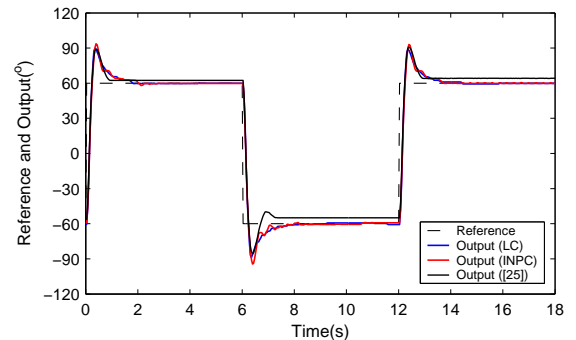


Fig. 9. Performance of INPC system (experiment).

errors. However, compared with the performance of the LCS (blue line), the performance of the INPC system is a little worse, due to the inevitable plant-model mismatch of the SMS resulting from the practical issues such as the static friction, dead zone, and measurement noise in real experiments. In addition, for the comparison with the INPC method, the NPC method in [25] is applied to the SMS. The experimental result is shown in Fig. 9 (black line). It can be seen that there exist steady-state output tracking errors due to the plant-model mismatch of the SMS in practical experiments.

Finally, it can be seen from Fig. 7 that, although the RTT delays of the Internet randomly vary between 4 steps and 7 steps, most of them keep on 4 steps. To further test the performance of the INPC method under more serious conditions, additional random packet dropouts with a rate of 30% shown in Fig. 10(a) are imposed on the data transmission over the Internet between the NIB and the NCB, where 1 and 0 denote the success and failure in the data transmission, respectively. Together with the real NIB-to-NCB Internet, the additional 30% packet dropouts result in the random RTT delays in Fig. 10(b). With these RTT delays, the experimental result is shown in Fig. 10(c). It can be seen that the INPC system still gives a satisfactory performance with zero steady-state errors, which is similar to that of the INPC system without additional packet dropouts shown in Fig. 9 (red line).

VI. CONCLUSIONS

This paper has presented an incremental networked predictive control method for the output tracking problem of NCSs based on the input-output difference equation model. An incremental model is used to predict the future output tracking errors of the control plant, and an incremental error feedback control law is employed to compute the future control commands, in such a way to actively compensate for the network-induced delay, packet disorder, and packet dropout in the feedback and forward channels.

The most appealing advantage of the proposed INPC method is that, no matter whether or not there exists a plant-model mismatch, the INPC system can achieve a zero steady-state output tracking error for the step reference signal, as long as the closed-loop system is stable. In addition, for the plant-model match case, the INPC system can provide the same output tracking performance and closed-loop stability as the corresponding LCS, and thus the controller design can follow the design procedure of the LCS. Simulation and experimental results have been given to demonstrate the effectiveness and applicability of the proposed method.

It is worth noting that, in this paper, only the ideal plant-model mismatch (i.e., time-invariant modeling error) has been considered for the design of INPC systems, and only the input constraints for the plant-model match case have been tested. However, most practical control systems are dynamical systems with time-varying parameter uncertainties and with various input and output constraints. The proposed INPC scheme in this paper is currently being studied for the above practical systems, although there still exist various challenging issues.

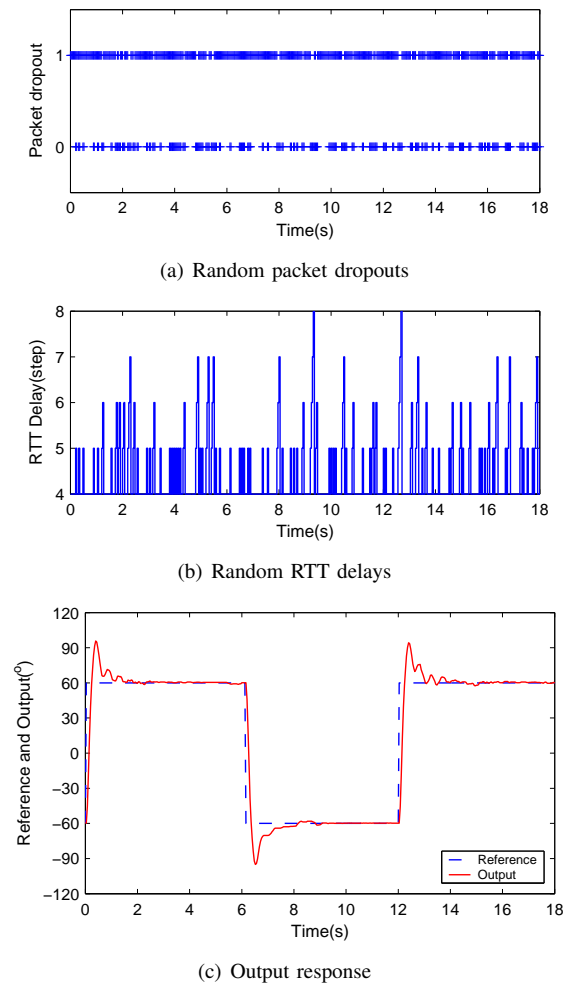


Fig. 10. Performance of INPC system with additional packet dropouts (experiment).

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