Consensus and Stability Analysis of Networked Multi-agent Predictive Control Systems

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Abstract -- This paper is concerned with the consensus and stability problem of multi-agent control systems via networks with communication delays and data loss. A networked multi-agent predictive control scheme is proposed to achieve output consensus and also compensate for the communication delays and data loss actively. The necessary and sufficient conditions of achieving both consensus and stability of the closed-loop networked multi-agent control systems are derived. An important result that is obtained is that the consensus and stability of closed-loop networked multi-agent predictive control systems are not related to the communication delays and data loss. An example illustrates the performance of the networked multi-agent predictive control scheme.

Keywords**:** Multi-agent systems, networked control systems, networked predictive control, consensus and stability.

I. INTRODUCTION

A multi-agent system is a system that is composed of several agents which can act in their own environments and coordinate with each other. Multi-agent technology has the following features: autonomy, distribution, coordination, self-organizing ability, learning ability and reasoning ability [1]. Multi-agent systems can solve practical problems with strong robustness, good reliability and high efficiency. There exist various multi-agent systems in real life, such as multi-robot systems, multi-satellite systems, air vehicle fleet, autonomous underwater vehicle queue and so on. In the past decade, the coordinated control technology has been studied extensively. The research work includes multi-agent consensus, formation of multi-agents, as well as coordination of autonomous control.

Similar to the synchronization in complex networks [2], the consensus of multi-agent systems plays a very important role in coordinative control of agents and there is a great amount of research work on this issue that has been done in recent years [3, 4]. A theoretical framework of the multi-agent consensus problem was proposed by Olfati–Saber and Murray [5]. For the case where the topology of multi-agents changes, the necessary and sufficient condition of the system consensus has been derived, which is there exists a spanning tree in the directed graph of the system [6]. The convergence of various consensus strategies has been studied for different multi-agent systems, for example, agents with first-order dynamics [7], agents with double-integrator dynamics [8, 9], and agents with immeasurable dynamical states [10, 11]. To design control protocols such that a set of agents can achieve consensus using available information, the existence of consensus protocols (*i.e.*, consensusability) has been discussed in [12, 13].

With the rapid development of network technology and Internet technology, more and more multi-agent systems communicate with each other via networks to achieve the mutual exchange of information between agents. This leads to a new system, called a networked multi-agent systems, such as networked control systems, Internet of things [14]. Control of networked multi-agent systems is a very complex task. This is mainly because communication, control, and information processing in a networked multi-agent system are distributed and there also exist communication constraints. So, it requires a coordinated control system for unified coordination and management to make multi-agents work together. In the past decade, various problems of networked multi-agent systems have been considered. The maximum network delay that can be tolerated by networked multi-agent systems has been studied in [5]. Asymptotic average consensus problem for multi-agent systems with time-varying delay has been addressed in [15, 16]. The consensus problem for Markovian jump second-order multi-agent systems with random communication delay has been discussed using stochastic switching topology in [17]. The global bounded consensus problem of networked multi-agent nonlinear systems with nonidentical node dynamics and communication time delay has been considered in [18]. Based on the networked predictive control method proposed in [19], a consensus protocol for discrete-time networked multi-agent systems has been presented to compensate for transmission delay [20].

Most current research work of multi-agent system mainly focuses on the consensus analysis. Actually, a multi-agent system can reach consensus, but it does not mean that the stability of the multi-agent system is guaranteed. For practical applications of multi-agent systems, both the consensus and stability should simultaneously be considered in the system design. To author's knowledge, very little research work has been paid to this so far. This paper combines both the consensus analysis and stability analysis together and obtains necessary and sufficient conditions of the system consensus and stability. To compensate for communication delay and data loss in networked multi-agent systems, a networked multi-agent predictive control scheme is proposed.

II. NETWORKED MULTI-AGENT PREDICTIVE CONTROL SCHEME

With the development of communication technology, multi-agents are integrated via networks in practice. There are various structures of networked multi-agent control systems in terms of the location of networks in a system. The most popular one is the networks are located between the sensors and controllers, for example, the formation of satellites that

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receive the position measurements from a communication network. The structure of networked multi-agent systems to be studied here is shown in Fig. 1.

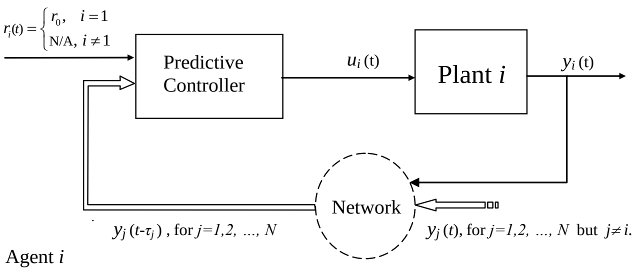

Fig. 1. The networked multi-agent predictive control system

To simplify the presentation of the proposed method in this paper, it is assumed that each agent can receive the output measurements of all the agents via networks, the delay caused by networks for the output of the i-th agent is bounded by d_i, the number of consecutive data loss on networks is bounded by c_i, the data transmitted through a network are with a time stamp, and all the agents in the system are synchronised.

Nowadays, most researchers in multi-agent systems assume the desired reference inputs of all the agents are zero. Actually, this is not the case in practice. To consider a more generic case, here it is assumed that the desired reference input of the first agent is given.

The communication topology of networked multi-agents is modeled by a digraph $\mathcal{G}=\{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, where $\mathcal{V}=\{1, 2, ..., N\}$ denotes the set of agents, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ the set of edges, and $\mathcal{A}=[a_{ij}]$ the nonnegative weighted adjacency matrix with $a_{ii}=0$. The directed edge $(i,j)\in\mathcal{V}$ means that the i-th agent can receive information from the j-th agent via networks. Adjacency element a_{ji} associated with edge (i,j) is positive. Let $l_{ii} = -\Sigma_{j=1}^{N} a_{ij}$.

The multi-agents to be considered are described as

$$\begin{aligned} x_i(t+1) &= A_i x_i(t) + B_i u_i(t) \\ y_i(t) &= C_i x_i(t) \end{aligned} \tag{1}$$

for $i=1, 2, ..., N,$ where $x_i \in \Re^{n_i}$, $y_i \in \Re^{l}$ and $u_i \in \Re^{m_i}$ are the state, output and input vectors of the i-th agent, respectively, and $A_i \in \Re^{n_i \times n_i}$, $B_i \in \Re^{n_i \times m_i}$ and $C_i \in \Re^{l \times n_i}$ are the system matrices. It is assumed that (A_i, C_i), $\forall i\in\mathcal{V}$, are observerable but the states of all the agents are immeasurable.

To prevent the data from dropout during data transmission from the i-th agent to other agents via networks, a data transmission strategy is adopted. In this strategy, the output data $[y_i(t), y_i(t-1),..., y_i(t-c_i)]$ at time t are sent from the i-th agent to other agents, which implies that the output data are always available if the number of consecutive data loss on networks is not greater than c_i.

From the assumptions, let $\tau_i=d_i+c_i$. As (A_i, C_i) is observerable, to estimate the state vector of an agent, the following state observer for the i-th agent is designed:

$$\begin{aligned} \hat{x}_i(t-\tau_i+1|t-\tau_i) &= A_i\hat{x}_i(t-\tau_i|t-\tau_i-1) + B_i u_i(t-\tau_i) + K_i^o\left(y_i(t-\tau_i) - \hat{y}_i(t-\tau_i|t-\tau_i-1)\right) \\ \hat{y}_i(t-\tau_i|t-\tau_i-1) &= C_i\hat{x}_i(t-\tau_i|t-\tau_i-1) \end{aligned} \tag{2}$$

where $\hat{x}_i(t-k\,|\,t-j) \in \Re^{n_i}$ $(k < j)$ denotes the state prediction of the i-th agent for time t-k using the information upto time t-j, $\hat{y}_i(.\,|\,.) \in \Re^{l}$ is the output prediction, and $K_i^o \in \Re^{n_i \times l}$ is the observer gain matrix.

Using the information available on the controller side, the states of the i-th agent from $t-\tau_i+2$ to time t can be predicted by

$$\begin{aligned} \hat{x}_i(t-\tau_i+k|t-\tau_i) &= A_i\hat{x}_i(t-\tau_i+k-1|t-\tau_i) + B_i u_i(t-\tau_i+k-1) \\ \hat{y}_i(t-\tau_i+k|t-\tau_i) &= C_i\hat{x}_i(t-\tau_i+k|t-\tau_i) \end{aligned} \tag{3}$$

for $k = 2, 3, ..., \tau_i$.

To track the desired reference input r_0, the following states are introduced:

$$z_1(t+1) = z_1(t) + \hat{y}_1(t|t-\tau_1) - r_0 \tag{4}$$

$$z_i(t+1) = z_i(t) + \hat{y}_i(t|t-\tau_i) - \hat{y}_1(t|t-\tau_1) \tag{5}$$

To compensate for the time delays and data loss caused by networks actively, the following predictive control protocol for networked multi-agents is proposed:

$$u_i(t) = K_i^y\hat{y}_i(t|t-\tau_i) + K_i^z z_i(t) + K_i^e\sum_{j=1}^{N} a_{ij}(\hat{y}_j(t|t-\tau_j) - \hat{y}_i(t|t-\tau_i)) \tag{6}$$

where $K_i^y \in \Re^{m_i \times l}$, $K_i^z \in \Re^{m_i \times m_i}$ and $K_i^e \in \Re^{m_i \times l}$ are the gain matrices to be designed, and a_{ij}, $\forall\, i,j\in\mathcal{V}$, are the elements of the nonnegative weighted adjacency matrix .

III. CONSENSUS AND STABILITY ANALYSIS OF NETWORKED MEULTI-AGENT PREDICTIVE CONTROL SYSTEMS

In multi-agent control systems, both the consensus and stability are the key issues. They are simultaneously analysed in this section.

Definition 1: Networked multi-agent control system (1) with control protocol (6) is input-output stable and achieves the output consensus if the following conditions hold:

1) $\lim_{t\to\infty}\|y_i(t)\| < \infty$, if $\|r_0\| < \infty$, for $t \geq 0$
2) $\lim_{t\to\infty}\|y_i(t) - y_j(t)\| = 0$

for $\forall i,j\in\mathcal{V}$, where r_0 is an constant reference input.

It is clear from Definition 1 that condition 1) defines the input-output stability and condition 2) does the output consensus. It defines how both stability and consensus of networked multi-agent control systems can be combined.

Replacing t by $t+\tau_i$ in observer (2) results in the following:

$$\begin{aligned} \hat{x}_i(t+1|t) &= A_i\hat{x}_i(t|t-1) + B_i u_i(t) + K_i^o(y_i(t) - \hat{y}_i(t|t-1)) \\ \hat{y}_i(t|t-1) &= C_i\hat{x}_i(t|t-1) \end{aligned} \tag{7}$$

Subtracting the observer state equation (7) from the multi-agent state equation (1) leads to the state error equation:

$$e_i(t+1) = (A_i - K_i^o C_i)e_i(t) \tag{8}$$

where $e_i(t) = x_i(t) - \hat{x}_i(t|t-1)$. Using the state prediction equation (3) recursively, the τ_i-th step ahead state prediction can be written as

$$\begin{aligned}\hat{x}_i(t|t-\tau_i) &= A_i^{\tau_i-1}\hat{x}_i(t-\tau_i+1|t-\tau_i) + \sum_{k=2}^{\tau_i} A_i^{\tau_i-k}B_i u_i(t+k-\tau_i-1) \\ &= A_i^{\tau_i-1}(A_i - K_i^o C_i)\hat{x}_i(t-\tau_i|t-\tau_i-1) + A_i^{\tau_i-1}K_i^o C_i x_i(t-\tau_i) \\ &\quad + \sum_{k=1}^{\tau_i} A_i^{\tau_i-k}B_i u_i(t+k-\tau_i-1)\end{aligned} \tag{9}$$

Employing (1) recursively results in

$$x_i(t) = A_i^{\tau_i}x_i(t-\tau_i) + \sum_{k=1}^{\tau_i} A_i^{\tau_i-k}B_i u_i(t+k-\tau_i-1) \tag{10}$$

Combining (9) and (10) yields

$$\begin{aligned}\hat{x}_i(t|t-\tau_i) &= A_i^{\tau_i-1}(A_i - K_i^o C_i)\hat{x}_i(t-\tau_i|t-\tau_i-1) + A_i^{\tau_i-1}K_i^o C_i x_i(t-\tau_i) \\ &\quad + x_i(t) - A_i^{\tau_i}x_i(t-\tau_i) \\ &= A_i^{\tau_i-1}(A_i - K_i^o C_i)(\hat{x}_i(t-\tau_i|t-\tau_i-1) - x_i(t-\tau_i)) + x_i(t) \\ &= x_i(t) - A_i^{\tau_i-1}(A_i - K_i^o C_i)e_i(t-\tau_i) \\ &= x_i(t) - A_i^{\tau_i-1}e_i(t-\tau_i+1)\end{aligned} \tag{11}$$

Using the above, control protocol (6) can be expressed as

$$\begin{aligned}u_i(t) &= K_i^y C_i x_i(t) + K_i^z z_i(t) + K_i^e \sum_{j=1}^{N} a_{ij}(C_j x_j(t) - C_i x_i(t)) \\ &\quad + K_i^e \sum_{j=1}^{N} a_{ij}(C_i A_i^{\tau_i-1}e_i(t-\tau_i+1) - C_j A_j^{\tau_j-1}e_j(t-\tau_j+1)) \\ &\quad - K_i^y C_i A_i^{\tau_i-1}e_i(t-\tau_i+1)\end{aligned} \tag{12}$$

Let $\Delta x_i(t) = x_i(t) - x_i(t-1)$ and $\Delta z_i(t) = z_i(t) - z_i(t-1)$. It is clear from (1) and (12) that the state increment of the i-th agent can be expressed by

$$\begin{aligned}\Delta x_i(t+1) &= A_i\Delta x_i(t) + B_i\Delta u_i(t) \\ &= A_i\Delta x_i(t) + B_iK_i^y C_i\Delta x_i(t) + B_iK_i^z\Delta z_i(t) \\ &\quad + B_iK_i^e \sum_{j=1}^{N} a_{ij}(C_j\Delta x_j(t) - C_i\Delta x_i(t)) \\ &\quad + B_iK_i^e \sum_{j=1}^{N} a_{ij}(C_iA_i^{\tau_i-1}\Delta e_i(t-\tau_i+1) - C_jA_j^{\tau_j-1}\Delta e_j(t-\tau_j+1)) \\ &\quad - B_iK_i^y C_iA_i^{\tau_i-1}\Delta e_i(t-\tau_i+1) \\ &= (A_i + B_i(K_i^y + l_{ii}K_i^e)C_i)\Delta x_i(t) + B_iK_i^z\Delta z_i(t) \\ &\quad + B_iK_i^e \sum_{j=1}^{N} a_{ij}C_j\Delta x_j(t) - B_iK_i^y C_iA_i^{\tau_i-1}\Delta e_i(t-\tau_i+1) \\ &\quad + B_iK_i^e \sum_{j=1}^{N} a_{ij}(C_iA_i^{\tau_i-1}\Delta e_i(t-\tau_i+1) - C_jA_j^{\tau_j-1}\Delta e_j(t-\tau_j+1))\end{aligned} \tag{13}$$

From (4), (5) and (11), $\Delta z_i(t+1)$ can be given by

$$\Delta z_1(t+1) = \Delta z_1(t) + C_1\Delta x_1(t) - C_1A_1^{\tau_1-1}\Delta e_1(t-\tau_1+1)$$

$$\begin{aligned}\Delta z_i(t+1) &= \Delta z_i(t) + \Delta\hat{y}_i(t|t-\tau_i) - \Delta\hat{y}_1(t|t-\tau_1) \\ &= \Delta z_i(t) + C_i\Delta x_i(t) - C_1\Delta x_1(t) \\ &\quad + C_1A_1^{\tau_1-1}\Delta e_1(t-\tau_1+1) - C_iA_i^{\tau_i-1}\Delta e_i(t-\tau_i+1)\end{aligned} \tag{14}$$

for $i = 2,3,\ldots, N$. Let

$$\Delta X(t) = \begin{bmatrix}\Delta x_1^T(t) & \Delta x_2^T(t) & \cdots & \Delta x_N^T(t)\end{bmatrix}^T$$

$$\Delta Z(t) = \begin{bmatrix}\Delta z_1^T(t) & \Delta z_2^T(t) & \cdots & \Delta z_N^T(t)\end{bmatrix}^T$$

$$E(t) = \begin{bmatrix}e_1^T(t-\tau_1+1), e_1^T(t-\tau_1), e_2^T(t-\tau_2+1), e_2^T(t-\tau_2), \cdots, e_N^T(t-\tau_N)\end{bmatrix}^T$$

Then, the compact forms for (13), (14) and (8) can be given by

$$\Delta X(t+1) = (A_D + B_{kyec} + B_{akc})\Delta X(t) + B_{kz}\Delta Z(t) + P(t) \tag{15}$$

$$\Delta Z(t+1) = (C_D - J_N \otimes C_1)\Delta X(t) + \Delta Z(t) + Q(t) \tag{16}$$

$$E(t+1) = A_{koc}E(t) \tag{17}$$

where

$$A_D = \text{diag}\{A_1, A_2, \cdots, A_N\}$$

$$\begin{aligned}A_{koc} = \text{diag}\{&A_1 - K_1^oC_1, A_1 - K_1^oC_1, A_2 - K_2^oC_2, A_2 - K_2^oC_2, \\ &\cdots, A_N - K_N^oC_N, A_N - K_N^oC_N\}\end{aligned}$$

$$\begin{aligned}B_{kyec} = \text{diag}\{&B_1(K_1^y + l_{11}K_1^e)C_1, B_2(K_2^y + l_{22}K_2^e)C_2, \cdots \\ &\cdots, B_N(K_N^y + l_{NN}K_N^e)C_N\}\end{aligned}$$

$$B_{akc} = \begin{bmatrix} a_{11}B_1K_1^eC_1 & a_{12}B_1K_1^eC_2 & \cdots & a_{1N}B_1K_1^eC_N \\ a_{21}B_2K_2^eC_1 & a_{22}B_2K_2^eC_2 & \cdots & a_{21}B_2K_2^eC_N \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1}B_NK_N^eC_1 & a_{N1}B_NK_N^eC_2 & \cdots & a_{NN}B_NK_N^eC_N \end{bmatrix}$$

$$B_{kz} = \text{diag}\{B_1K_1^z, B_2K_2^z, \cdots, B_NK_N^z\}$$

$$C_D = \text{diag}\{C_1, C_2, \cdots, C_N\}$$

$$P(t) = \begin{bmatrix}p_1^T(t), p_2^T(t), \cdots, p_N^T(t)\end{bmatrix}^T$$

$$Q(t) = \begin{bmatrix}q_1^T(t), q_2^T(t), \cdots, q_N^T(t)\end{bmatrix}^T$$

$$\begin{aligned}p_i(t) &= B_iK_i^e \sum_{j=1}^{N} a_{ij}C_jA_j^{\tau_j-1}(e_j(t-\tau_j) - e_j(t-\tau_j+1)) \\ &\quad - l_{ii}B_iK_i^eC_iA_i^{\tau_i-1}(e_i(t-\tau_i+1) - e_i(t-\tau_i)) \\ &\quad - B_iK_i^yC_iA_i^{\tau_i-1}(e_i(t-\tau_i+1) - e_i(t-\tau_i))\end{aligned}$$

$$q_1(t) = C_1A_1^{\tau_1-1}(e_1(t-\tau_1) - e_1(t-\tau_1+1))$$

$$\begin{aligned}q_i(t) &= C_1A_1^{\tau_1-1}(e_1(t-\tau_1+1) - e_1(t-\tau_1)) \\ &\quad - C_iA_i^{\tau_i-1}(e_i(t-\tau_i+1) - e_i(t-\tau_i)), \text{ for } i = 2,3,\ldots,N\end{aligned}$$

$$J_N = \begin{bmatrix}[0,1,\cdots,1]^T & 0_{n\times(n-1)}\end{bmatrix}$$

and $\otimes$ denotes the Kronecker product of matrices.

Thus, it is clear from (15), (16) and (17) that the closed-loop networked multi-agent predictive control system can be described by the following compact form:

$$\begin{bmatrix} \Delta X(t+1) \\ \Delta Z(t+1) \\ E(t+1) \end{bmatrix} = \begin{bmatrix} A_D + B_{kyec} + B_{akc} & B_{kz} & \alpha \\ C_D - J_N \otimes C_1 & \mathbf{I} & \beta \\ \mathbf{0} & \mathbf{0} & A_{koc} \end{bmatrix} \begin{bmatrix} \Delta X(t) \\ \Delta Z(t) \\ E(t) \end{bmatrix} \tag{18}$$

where $\alpha E(t) = P(t)$, $\beta E(t) = Q(t)$, and ***0*** and ***I*** denote a zero matrix and identity matrix with an appropriate dimension, respectively. Let

$$\Omega = \begin{bmatrix} A_D + B_{kyec} + B_{akc} & B_{kz} \\ C_D - J_N \otimes C_1 & \mathbf{I} \end{bmatrix} \tag{19}$$

It is clear from the above that if system (18) is stable then $\Delta x_i(t) \to 0$, $\Delta z_i(t) \to 0$ and $e_i(t) \to 0$ as $t \to \infty$, for *i=1, 2, …, N*. From $e_i(t) \to 0$ as $t \to \infty$, it leads to

$$\hat{x}_i(t|t-\tau_i) \to x_i(t) \text{ as } t \to \infty \text{ (from (11))}$$

So,

$$\hat{y}_i(t|t-\tau_i) \to y_i(t) \text{ as } t \to \infty \tag{20}$$

because of (3). Eq. (4) ad (5) can be rewritten as

$$\Delta z_1(t+1) = \hat{y}_1(t|t-\tau_1) - r_0 \tag{21}$$

$$\Delta z_i(t+1) = \hat{y}_i(t|t-\tau_i) - \hat{y}_1(t|t-\tau_1) \tag{22}$$

for *i = 2,3,…, N*. From $\Delta z_i(t) \to 0$ as $t \to \infty$, $\forall i \in \mathcal{V}$, it implies from (19) and (20) that

$$\hat{y}_1(t|t-\tau_1) \to r_0 \text{ as } t \to \infty \tag{23}$$

$$\hat{y}_i(t|t-\tau_i) \to \hat{y}_1(t|t-\tau_1) \to y_1(t) \text{ as } t \to \infty, \text{ for } i=2, 3, \dots, N \tag{24}$$

Thus, it can be concluded from (20), (23) and (24) that

$$y_1(t) \to r_0 \text{ as } t \to \infty \tag{25}$$

$$y_i(t) \to y_1(t) \text{ as } t \to \infty, \text{ for } i=2, 3, \dots, N \tag{26}$$

Clearly, the two conditions of Definition 1 are satisfied. In other words, the closed-loop networked multi-agent predictive control system is stable and also achieves the consensus. It is well-known that the necessary and sufficient stability conditions of system (18) are all the eigenvalues of matrices Ω and $A_i - K_i^o C_i$, $\forall i \in \mathcal{V}$, are within the unit circle. So, summarising the above results gives the following theorem:

Theorem 1: The networked multi-agent control system (1) with control protocol (6) is stable and achieves consensus if and only if all the eigenvalues of matrices Ω and $A_i - K_i^o C_i$, $\forall i \in \mathcal{V}$, are within the unit circle.

It is also noted from the above theorem that both the consensus and stability of the closed-loop networked multi-agent predictive control systems are not related to network delays. This is a significant achievement for networked multi-agent control systems. Actually, after the observers (7) converge, the networked multi-agent predictive control system has the same control performance as the multi-agent control system without communication delays and data loss (*i.e.*, $\tau_i=0$, $\forall i \in \mathcal{V}$,).

If there do not exist network delays or data loss, or the outputs of all the agents are directly measured without networks, this means that $\tau_i=0$, $\forall i \in \mathcal{V}$. For this case, the control protocol (6) can be modified to be

$$u_i(t) = K_i^y y_i(t) + K_i^z z_i(t) + K_i^e \sum_{j=1}^{N} a_{ij}(y_j(t) - y_i(t)) \tag{27}$$

for *i=1, 2, …, N*, where

$$z_1(t+1) = z_1(t) + y_1(t) - r_0 \tag{28}$$

$$z_i(t+1) = z_i(t) + y_i(t) - y_1(t) \tag{29}$$

for *i=2, 3, …, N*. Following the similar procedure used for the case of $\tau_i \neq 0$, $\forall i \in \mathcal{V}$, the closed-loop multi-agent control system without networks can be described by

$$\begin{bmatrix} \Delta X(t+1) \\ \Delta Z(t+1) \end{bmatrix} = \Omega \begin{bmatrix} \Delta X(t) \\ \Delta Z(t) \end{bmatrix} \tag{30}$$

where matrix Ω is the same one given by (19). Thus, the necessary and sufficient conditions of networked multi-agent control system (1) with control protocol (27) being stable and achieving consensus are all the eigenvalu*es* of matrix Ω are within the unit circle.

IV. AN EXAMPLE

This section uses an example to demonstrate how the networked multi-agent predictive control scheme proposed in this paper works. Three different agents are considered with the following system matrices:

$$A_1 = \begin{bmatrix} 1.7 & -1.3 \\ 1.6 & -1.8 \end{bmatrix}, B_1 = \begin{bmatrix} 1.0 \\ 2.0 \end{bmatrix}, C_1 = \begin{bmatrix} 1.0 \\ 0.3 \end{bmatrix}^T$$

$$A_2 = \begin{bmatrix} 1.8 & -1.4 \\ 1.8 & -1.9 \end{bmatrix}, B_2 = \begin{bmatrix} 1.7 \\ 3.4 \end{bmatrix}, C_2 = \begin{bmatrix} 0.7 \\ 0.2 \end{bmatrix}^T$$

$$A_3 = \begin{bmatrix} 1.4 & -1.1 \\ 1.3 & -1.5 \end{bmatrix}, B_3 = \begin{bmatrix} 0.8 \\ 1.6 \end{bmatrix}, C_3 = \begin{bmatrix} 1.1 \\ 0.4 \end{bmatrix}^T$$

The three agents are communicated via a network with a structure in which there is a one-to-one connection between them, *i.e.*, the elements of the nonnegative adjacency weighted matrix are $a_{12}=a_{21}=a_{13}=a_{31}=a_{23}=a_{32}=1$ and $a_{11}=a_{22}=a_{33}=0$.

There are many methods to design the gain matrices in predictive control protocol (6). Here, following the eigenstructure assignment method [21], one of possible solutions for the control gain matrices that make all the eigenvalues of matrix Ω be within the unit circle is obtained, which results from the following control gains:

$$K_i^y = 0.25, K_i^z = -0.20, K_i^e = -0.15$$

for *i=1, 2, 3*.

Using the pole assignment method, the observer gain matrices for the three agents are designed as

$$K_1^o = \begin{bmatrix} -0.1700 \\ -0.7666 \end{bmatrix}, K_2^o = \begin{bmatrix} -0.2759 \\ -1.0345 \end{bmatrix}, K_3^o = \begin{bmatrix} -0.1535 \\ -0.5780 \end{bmatrix}$$

to assign the desired poles of the three observers to 0.1 and 0.2. In this example, there are the following assumptions:

1) The communication delays are $d_1=3$, $d_2=2$, $d_3=4$ and the numbers of the consecutive data loss in the network for individual agents are $c_1=1$, $c_2=3$, $c_3=2$.
2) The initial values of all the agent states, control inputs and observer states are zero. The desired reference input for the first agent is $r_0=1$ for $0<t\leqslant 50$ and $r_0=2$ for $t>50$.

From the first assumption in the above, it can be calculated that $\tau_1= 4$, $\tau_2= 5$, $\tau_3= 6$. Three cases are discussed in this example.

Case 1

For this case, there do not exist communication delays or data loss in the networked three-agent control system, and the control protocol (27) are utilised, *i.e.*,

$$u_i(t) = K_i^y y_i(t) + K_i^z z_i(t) + K_i^e \sum_{j=1}^{3} a_{ij}(y_j(t)-y_i(t))$$

for $i=1, 2, 3$, where $z_1(t+1) = z_1(t) + y_1(t) - r_0$ and $z_i(t+1) = z_i(t) + y_i(t) - y_1(t)$, for $i=2, 3$. The outputs $y_i(t)$ of the three agents are shown in Figure 2. It is clear that the closed-loop three-agent control systems are stable and track the desired reference input very well.

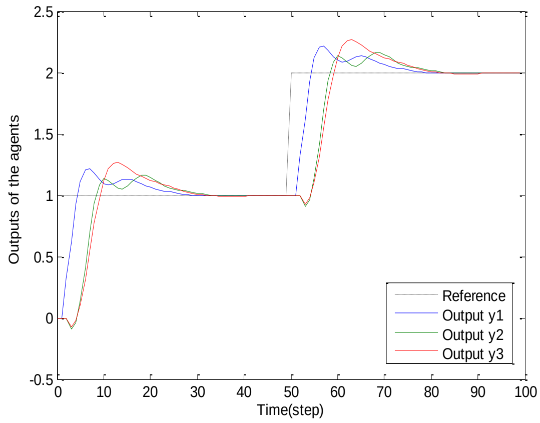

Figure 2 The outputs of the agents (Case 1)

Case 2

For this case, there are communication delays and data loss in the networked three-agent control system, and a normal control protocol without compensating for network delays and data loss is used as follows:

$$u_i(t) = K_i^y y_i(t-\tau_i) + K_i^z z_i(t) + K_i^e \sum_{j=1}^{3} a_{ij}(y_j(t-\tau_j)-y_i(t-\tau_i))$$

for $i=1, 2, 3$, where $z_1(t+1) = z_1(t) + y_1(t-\tau_1) - r_0$ and $z_i(t+1) = z_i(t) + y_i(t-\tau_i) - y_1(t-\tau_1)$, for $i=2, 3$. In this example, if one of τ_1, τ_2 and τ_3 is not zero, the closed-loop networked three-agent control system with the above control protocol is unstable. For example, for $\tau_1=0$, $\tau_2=0$ and $\tau_3=1$, the outputs $y_i(t)$ of the three agents are shown in Figure 3. Clearly, the closed-loop system is unstable. This is mainly because the communication delays and data loss are not compensated in the control protocol.

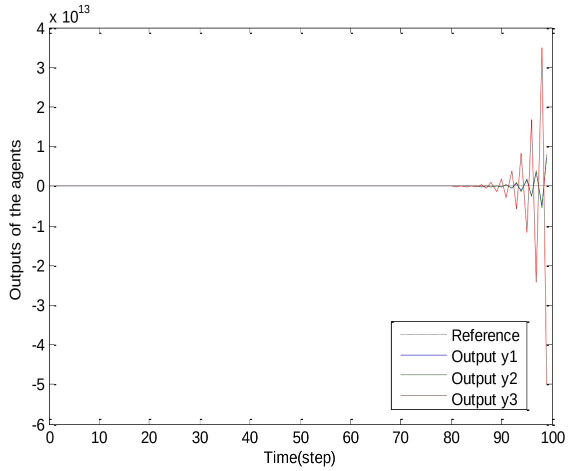

Figure 3 The outputs of the agents (Case 2)

Case 3

For this case, there are communication delays and data loss in the networked three-agent control system, and the networked multi-agent predictive control scheme is employed. The control protocol is given by (6), *i.e.*,

$$u_i(t) = K_i^y \hat{y}_i(t|t-\tau_i) + K_i^z z_i(t) + K_i^e \sum_{j=1}^{3} a_{ij}(\hat{y}_j(t|t-\tau_j)-\hat{y}_i(t|t-\tau_i))$$

for $i=1, 2, 3$, where $z_1(t+1) = z_1(t) + \hat{y}_1(t\,|\,t-\tau_1) - r_0$ and $z_i(t+1) = z_i(t) + \hat{y}_i(t|t-\tau_i) - \hat{y}_1(t|t-\tau_1)$, for $i=2, 3$. The outputs $y_i(t)$ of the three agents are shown in Figure 4. It can be noted from the simulation results that the performance of the closed-loop networked three-agent control system is very similar to the one of the system without communication delay and data loss.

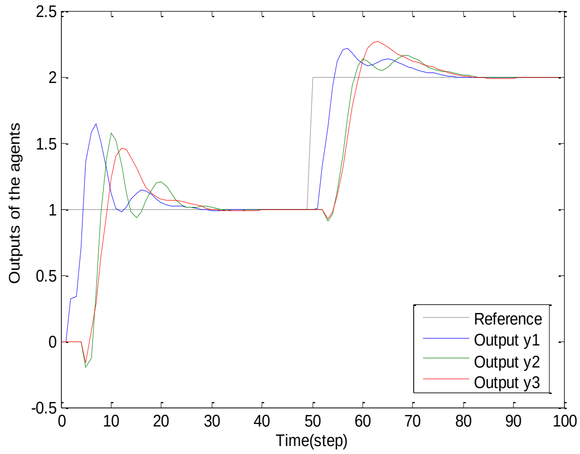

Figure 4 The outputs of the agents (Case 3)

For comparison, the output errors between Case 3 and Case 1 are shown in Figure 5. For $0\leqslant t\leqslant 50$, there exist the output errors between Case 3 and Case 1, which is caused by the observers. When the reference input changes from $r_0=1$ to $r_0=2$ at $t=50$, the output errors between Case 3 and

Case 1 are zero because the observers of the agents converge. It implies that the performance of the networked multi-agent predictive control systems with communication delays and data loss is the same as the one of the system without network. This shows the networked multi-agent predictive control scheme proposed in this paper actively compensates for communication delays and data loss completely.

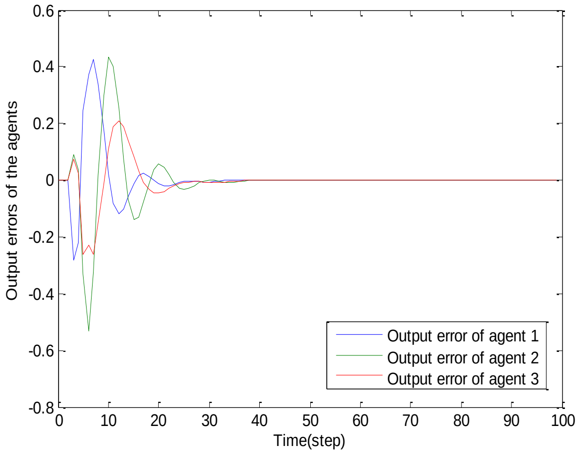

Figure 5 The output errors between Case 3 and Case 1

V. CONCLUSIONS

This paper has studied both consensus and stability of networked multi-agent control systems with communication delays and data loss. To compensate for communication delays and data loss actively, the networked multi-agent predictive control scheme has been proposed to achieve output consensus. The output consensus and input-output stability analysis has provided the necessary and sufficient conditions of achieving both consensus and stability of the closed-loop networked multi-agent predictive control systems. It has also been concluded that the consensus and stability of closed-loop networked multi-agent predictive control systems are not related to communication delays and data loss. The example has illustrated that the performance of the networked multi-agent predictive control system is the same as the one of the multi-agent control system without network.

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